Date: 12/10/2021

Q1. If $T_n = (-1)^{n-1} \cdot n^3$. Find T_9 term of the A.P.

Q2. If $T_n = (n-1)(2-n)$. Then find T_1, T_2, and T_3.

Q3. Find 440^{th} and 441^{st} terms of the sequence defined by

$$T_n = \begin{cases} \dfrac{n}{\dfrac{n}{96}-1}, \text{ if } n \text{ is not the square of a natural number} \\ \dfrac{13}{2}, \text{ If } n \text{ is the square of a natural number} \end{cases}$$

Q4. If $T_n = \begin{cases} 1 & \text{If } n \text{ is an odd integer} \\ -1 & \text{If } n \text{ is an even integer} \end{cases}$

Find the T_7 and T_8 term of the above sequence.

Q5. Find the 19^{th} and 20^{th} terms of the sequence defined by

$$T_n = \begin{cases} n^2, & \text{when } n \text{ is even} \\ n^2+1 & \text{when } n \text{ is odd} \end{cases}$$

Q6. If the n^{th} term of a progression is a linear expression in n, then prove that it is an A.P.

Q7. Write the first five terms of the sequence defined by the following:

$$a_n = n(n+2).$$

Q8. If S_n denotes the sum of first n terms of an A.P., show that its common difference is $S_n - 2S_{n-1} + S_{n-2}$.

Q9. If the sum of 101 terms of an A.P. is 1212, then find the middle term of that A.P..

Q10. Four numbers are in A.P such that the sum of first and last term is 8 and the product of both middle terms is 15, then find the the least number of the series.

Q11. If $a, \frac{1}{b}, c$ and $\frac{1}{p}, q, \frac{1}{r}$ forms two arithmatic progression of same common difference such that a, q, c are in A.P., prove that $\frac{1}{p}, \frac{1}{b}, \frac{1}{r}$ are also in A.P.

Q12. If a, b, c are in A.P., then prove that $a^3 + c^3 - 8b^3 = -6abc$.

Q13. If $a, b, c \in R^+$ form an A.P. then prove that $\left(a + \frac{1}{bc}\right), \left(b + \frac{1}{ac}\right), \left(c + \frac{1}{ab}\right)$ are also in A.P.

Q14. How many terms are there in the A.P.: $14\frac{1}{2}$, 12, ..., – 38?

Q15. The 5^{th} and 10^{th} terms of an A.P. are 13 and 28 respectively. Find this A.P. and obtain its 20^{th} term.

Q16. Write the first five terms of the sequence defined by the following:

$$a_n = 2^n.$$

Q17. Write the first five terms of the sequence defined by the following:

$$a_n = \frac{n}{n+1}.$$

Q18. Find the indicated terms in each of the following sequence:

$$a_n = 4n - 3, \ a_{17}, \ a_{24}.$$

Q19. Write the first five terms of the sequence defined by the following:

$$a_n = \frac{n(n^2+5)}{4}.$$

Q20. Write the first five terms of the sequence defined by the following:

$$a_n = (-1)^{n-1} 5^{n+1}.$$

Q21. Write the first five terms of the sequence defined by the following:

$$a_n = \frac{2n-3}{6}.$$

Q22. Find the indicated terms in each of the following sequence:

$$a_n = \frac{n(n-2)}{n+3}; \quad a_{20}.$$

Q23. Find the indicated terms in each of the following sequence:

$$a_n = (-1)^{n-1} n^3, \ a_9.$$

Q24. Find the indicated terms in each of the following sequence:

$$a_n = \frac{n^2}{2^n}, \ a_7.$$

Q25. In an A.P. if m^{th} term is n and the n^{th} term is m, where $m \neq n$, find the p^{th} term.

Q26. What is the 20^{th} term of the sequence defined by

$$a_n = (n-1)(2-n)(3+n)?$$

Q27. If 7^{th} times 7^{th} term of an A.P. is equal to 11^{th} times it's 11^{th} term. Show that the 18^{th} term of that A.P. is zero.

Q28. The n^{th} term of a progression is $(2n + 3)$. Prove that it is an A.P. Find its 10^{th} term.

Q29. Find the first four terms of the sequence whose n^{th} term is given as

$$T_n = (-1)^{n-1} \cdot \cos\frac{n\pi}{4}$$

Q30. Write the first four terms of the given sequence, whose n^{th} term is given by,

$$T_n = (-1)^n \cdot \sin\frac{n\pi}{2}$$

Q31. Find first five terms of the sequence defined by

$$a_1 = 1, \quad a_n = a_{n-1} + 3 \quad \text{for} \quad n \geq 2$$

Q32. Which term of the A.P.: (a) 5, 8, 11, ... is 320; (b) 4, 9, 14, ... is 114.

Q33. Find the first six terms of the sequence, if $T_1 = -5$, $T_n = \frac{T_{n-1}}{n+1}$, $n \geq 2$.

Q34. If the sum of n terms of an A.P. is $cn(n-1)$, where $c \neq 0$, then find the sum of squares of these terms.

Q35. Find the maximum sum of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + ...$

Q36. The 4th term of an A.P. is three times the first and the seventh term exceeds twice the third term by 1. Find the first term and the common difference.

Q37. In an A.P. of 99 terms, the sum of all the odd numbered terms is 2550. Then find the sum of all 99 terms of the A.P.

Q38. If the m^{th} term of an A.P. be $\left(\frac{1}{n}\right)$ and n^{th} term be $\left(\frac{1}{m}\right)$, Show that it's $(mn)^{th}$ term is 1.

Q39. Which term of an A.P. $7 - 4i, 6 - 2i, 5 - 0i, 4 + 2i$, ... is (i) purely real (ii) purely imaginary?

Q40. Which term of the A.P.: 121, 117, 113, ... is the first negative term?

O41. If the 9th term of an A.P. is zero. Prove the 29th term is double the 19th term.

Q42. Find the 2nd and the r^{th} terms of an A.P. whose 6th and 8th terms are 12 and 22 respectively.

Q43. How many terms are there in the A.P.: (a) 10, 13, 16, ... 49; (b) 7, 13, 19, ... 199.

Q44. Find the value of K so that

(a) $2K + 1, 2K - 1, 3K + 4$ are in A.P. (b) $3K - 1, K + 1, K + 3$ are in A.P.

Q45. If m times the m^{th} term of an A.P. is equal to n times its n^{th} term show that the $(m + n)^{th}$ term of the A.P. is zero.

Q46. Find the first five terms of the sequence defined by.

$$a_1 = -1,\ a_n = \frac{a_{n-1}}{n},\ n \geq 2$$

Q47. How many two digit numbers leave the remainder 1 when divided by 5.

Q48. If the sum of three numbers which are in A.P. is 27 and product of first and last is 77, then find the numbers.

Q49. Prove that a, b, c are in A.P. if and only if $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.

Q50. For an A.P. show that $T_p + T_{p+2q} = 2T_{p+q}$.

Q51. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respetively, find the 25th term.

Q52. If the sides of a right angled triangle are in A.P. then find the sines of their acute angles.

Q53. Find the sum of first 24 terms of the A.P. a_1, a_2, a_3, ... if it is known that
$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

Q54. In an A.P. if $S_1 = T_1 + T_2 + ... + T_n$ (n odd), $S_2 = T_2 + T_4 + ... + T_{n-1}$. Then find the value of S_1/S_2 in terms of n.

Q55. The sum of certain number of terms in A.P. is 5500. If first and the last terms are 100 and 1000 respectively. Find the number of terms.

Q56. Find the sum of n terms of a sequence whose n^{th} term is given by $t_n = 3n + 4$.

Q57. If $S_n = 2n^2 + 3n$ denotes the sum of n terms of progression, prove that it is an A.P. Find its r^{th} term.

Q58. If the sum of n terms of a progression be a quadratic expression in n. Show that it is an A.P.

Q59. If the sum of first n, $2n$, $3n$ terms of an A.P. be S_1, S_2 and S_3 respectively, then prove that $S_3 = 3(S_2 - S_1)$.

Q60. Write down the first five terms of each of the following sequences, whose n^{th} terms are:

(a) $T_n = (-1)^{n-1} \cdot 5^{n+1}$ (b) $T_n = \dfrac{n(n^2+5)}{4}$

Q61. If the ratio between the sum of n terms of two A.P. is $(3n + 8) : (7n + 15)$, then find the ratio between their 12^{th} term.

Q62. Write the next five terms of the given sequences.

(a) $a_1 = a_2 = 2,\ a_n = a_{n-1} - 1\ (n > 2)$ (b) $a_1 = a_2 = 1,\ a_n = a_{n-1} + a_{n-2}\ (n > 2)$

Find $\dfrac{a_{n+1}}{a_n}$ for $n = 1, 2, 3, 4, 5$

Q63. Write the first five terms of the given sequence, using the given rule, in each case the initial value of index is 1.

(a) $T_n = 2n^2 - n + 1$ (b) $T_n = \dfrac{n^n}{3^n}$

Q64. If a, b, c are in A.P., prove that $a^3 + 4b^3 + c^3 = 3b(a^2 + c^2)$.

Q65. If $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P., show that either a, b, c are in A.P. or $ab + bc + ac = 0$.

Q66. If, $a_1, a_2, a_3, \ldots\ldots\ a_n$ be an A.P. of non-zero terms, prove that $\dfrac{1}{a_1 a_2} + \dfrac{1}{a_2 a_3} + \ldots\ldots + \dfrac{1}{a_{n-1} a_n} = \dfrac{n-1}{a_1 a_n}$

Q67. In an A.P. of n terms, prove that the sum of k^{th} term from the beginning and k^{th} term from the end is independent of k and equals the sum of first and last terms.

Q68. Write first four terms in each of the sequences

(a) $T_n = \left(\dfrac{2n-3}{4}\right)$ (b) $T_n = (-1)^{n-1} \cdot 2^{n+1}$

Q69. Write the first five terms of the sequence using the given rule, in each case the initial value of index is 1.

(a) $T_n = \dfrac{(-1)^{n-1}}{n^3}$ (b) $T_n = \left(\dfrac{1}{2}\right)^{2n+1} + \left(-\dfrac{1}{2}\right)^{2n}$

Q70. If the p^{th}, q^{th} and r^{th} terms of an A.P. are a, b and c respectively, then show that $a(q - r) + b(r - p) + c(p - q) = 0$.

Q71. In an A.P. if $S_n = n^2p$ and $S_m = m^2p$, $m \neq n$ then prove that $S_p = p^3$.

Q72. Prove that the sum of the latter half of $2n$ terms of any A.P. is one third the sum of 3n terms of the same A.P.

Q73. In an A.P. if $S_p = q$ and $S_q = p$, prove that $S_{p+q} = -(p + q)$.

Q74. In an A.P. whose first term is a, if the sum of first p terms is zero, show that the sum of next q terms is $-\dfrac{a(p+q)}{p-1}q$.

Q75. If the sum of first p terms of an A.P. is equal to sum of its first q terms, show that the sum of its $p + q$ terms is zero.

Q76. Divide 32 into four parts which are in A.P. such that the ratio of product of extremes to the product of means is 7 : 15.

Q77. Find the middle term(s) in the A.P. 20, 16, 12, ..., – 176.

Q78. If a, b, c are in A.P. show that $\dfrac{1}{(\sqrt{b}+\sqrt{c})}, \dfrac{1}{(\sqrt{c}+\sqrt{a})}, \dfrac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

Q79. If the sum of n terms of an A.P. is $(pn + qn^2)$ where p and q are constants, find the common difference.

Q80. If a, b, c are in A.P. then prove that $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are also in A.P.

Q81. Consider two A.P.'s

S_1 : 2, 7, 12, 17, ... 500 terms

S_2 : 1, 8, 15, 22, ... 300 terms

Find the number of common terms. Also find the last common term.

Q82. Given two A.P.'s 2, 5, 8, 11, ... T_{60} and 3, 5, 7, 9, ... T_{50}, then find the number of terms which are identical.

Q83. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of sides of the polygon.

Q84. Write the first five terms of the sequences and obtain the corresponding series:

$$a_1 = 3, \quad a_n = 3a_{n-1} + 2, \quad \text{for all} \quad n > 1.$$

Q85. The sums of n terms of three A.P.s are S_1, S_2, S_3 and the first term of each is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Q86. Find the angles of a quadrilateral which are in A.P. having common difference 10°.

Q87. In an A.P. it is given that $T_{p+1} = 2T_{q+1}$, prove that $T_{3p+1} = 2T_{p+q+1}$.

Q88. In an A.P. the first term is 2 and the sum of first five terms is $1/4^{th}$ of the sum of next five terms. Show that the 20^{th} term is – 112 and the sum of first 20 terms is – 1100.

Q89. Consider an A.P., a_1, a_2, a_3, ... such that $a_3 + a_5 + a_8 = 11$ and $a_4 + a_2 = -2$. Then find the value of $a_1 + a_6 + a_7$.

Q90. If the ratio of sum to n terms of two A.P. is (5n + 3) : (3n + 4), then find the ratio of their 17th terms.

Q91. If $\log_2 (5 \times 2^x + 1)$, $\log_4 (2^{1-x} + 1)$ and 1 are in A.P. then find the value of x.

Q92. Find the sum of all three digit natural numbers, which are divisible by 7.

Q93. The digits of a positive integer, having three digits are in A.P. and their sum is 15. The number obtained by reversing the digit is 594 less than the original number find the number.

Q94. If the sum of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$, find the ratio of their 18^{th} terms.

Q95. Find the sum to n terms of the A.P. whose k^{th} term is $5k + 1$.

Q96. If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116, find the last term.

Q97. How many terms of the A.P. $-6, \frac{-11}{2}, -5$ ... are needed to give the sum -25?

Q98. Find the sum of all natural numbers lying between 100 and 1000, which are multiple of 5.

Q99. Find the sum of odd integers from 1 to 2001.

Q100 Write the first five terms of the sequences in the following and obtain the corresponding series:

$$a_1 = -1, \quad a_n = \frac{a_{n-1}}{n}, \quad n \geq 2.$$

Q101 Find the sum of all two digit numbers which, when divided by 4, yield 1 as remainder.

Q102 Find the sum of all numbers between 200 and 400 which are divisible by 7.

Q103 If the sum of three numbers in A.P. be 24 and their product is 440, find the numbers.

Q104 Show that the sum of $(m + n)^{th}$ and $(m - n)^{th}$ terms of an A.P. is equal to twice the m^{th} term.

Q105 Write the first three terms in each of the following sequences defined as:

(i) $a_n = 2n + 5$ (ii) $a_n = \frac{n-3}{4}$

Q106 If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.

Q107 If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P., prove that a, b, c are in A.P.

Q108 Let the sequence a_n be defined as follows:

$$a_1 = 1, \quad a_n = a_{n-1} + 2 \quad \text{for } n \geq 2.$$

Find first five terms and write corresponding series.

Q109 The sum of first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

Q110 If a, b, c, d and p are different real numbers such that

$$(a^2 + b^2 + c^2)\, p^2 - 2\,(ab + bc + cd)\, p + (b^2 + c^2 + d^2) \leq 0,$$

then show that a, b, c and d are in G.P.

Q111 If a, b, c are in A.P. prove that $\frac{ab+ac}{bc}, \frac{bc+ba}{ca}, \frac{ca+bc}{ab}$ are in A.P.

Q112 The ratio of the sum of m and n terms of an A.P. is $m^2 : n^2$ show that the ratio of m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

Q113 The ratio of the sums of n terms of two A.P.s is $(7n + 1) : (4n + 27)$ find the ratio of their 11^{th} terms.

Q114 If the sum of p, q and r terms of an A.P. be a, b and c respectively then prove that $\frac{a}{p}(q - r) + \frac{b}{q}(r - p) + \frac{c}{r}(p - q) = 0$.

Q115 The p^{th} term of an A.P. is a and q^{th} term is b. Prove that the sum of its $p + q$ terms is $\frac{p+q}{2}\left[a + b + \frac{a-b}{p-q}\right]$.

Q116 The first, second and the last terms of an A.P. are a, b, c respectively. Show that the sum of the A.P. is $\frac{(b + c - 2a)(a + c)}{2(b - a)}$.

Q117 If $S_1, S_2, S_3, \ldots S_m$ are the sums of n terms of m A.P.'s whose first terms are 1, 2, 3, ... m and common differences are 1, 3, 5, ..., $2m - 1$ respectively, then find the value of $\sum_{i=1}^{m} S_m$.

Q118 If $a_1, a_2, \ldots a_n$ are in A.P., where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Q119 If the m^{th} term of an A.P. is $1/n$ and n^{th} term is $1/m$. Show that the sum of mn terms is $\frac{mn+1}{2}$.

Q120 If, a_1, a_2, a_3, ... are in A.P. with common difference d, then prove that
$\sin d[\text{cosec } a_1 \text{ cosec } a_2 + \text{cosec } a_2 \text{ cosec } a_3 + \ldots + \text{cosec } a_{n-1} \text{ cosec } a_n] = \cot a_1 - \cot a_n$.

Q121 The series of n natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9), ... Show that the sum of numbers in the n^{th} group is $(n - 1)^3 + n^3$.

Q122 If a^2, b^2, c^2 are in A.P., then show that the following are also in A.P.

(a) $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$

(b) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$

Q123 How many terms of the A.P.

(a) 1, 4, 7, ... are needed to give the sum 590?

(b) 20, 18, 16, ... are needed to give the sum zero.

(c) 16, 14, 12, ... are needed to give the sum 60? Explain the double answer.

(d) 54, 51, 48, ... are needed to give the sum 513? Explain the double answer.

Q124 Find the sum of the series

(a) 5 + 13 + 21 + ... + 189

(b) 6 + 10 + 14 + ... + 106

(c) $1 + 2 + 3 + \ldots + n$

(d) $\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots + \log \frac{a^n}{b^{n-1}}$

Q125 Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Date: 12/10/2021

S1. Given, $T_n = (-1)^{n-1} \cdot n^3$

Putting, $n = 9$, we get

$$T_9 = (-1)^{9-1} \cdot 9^3$$
$$= (-1)^8 \cdot 729 = 729$$

S2. $\because$ $T_n = (n - 1)(2 - n)$

$\therefore$ $T_1 = (1 - 1)(2 - 1) = 0$

$$T_2 = (2 - 1)(2 - 2) = 0$$
$$T_3 = (3 - 1)(2 - 3) = -2$$

S3. Since, 440 is not a perfect square, therefore,

$$T_n = \frac{n}{\frac{n}{96} - 1} \Rightarrow T_{440} = \frac{440}{\frac{440}{96} - 1} = \frac{5280}{43}$$

Again, $441 = (21)^2$

Therefore, $T_{441} = \frac{13}{2}$

S4. $\because$ T_7 is a odd term of the sequence, hence

$$T_7 = 1$$

Similarly, T_8 is an even term of the sequence, hence

$$T_8 = -1.$$

S5. $T_{19} = 19^2 + 1,$ $\{\because$ 19 is odd$\}$

$= 362$

$T_{20} = 20^2 = 400.$ $\{\because$ 20 is even$\}$

S6. Since, $T_n = an + b$, where a and b are constants

$$T_{n-1} = a(n - 1) + b$$
$$T_n - T_{n-1} = (an + b) - [(a(n - 1) + b]$$
$$= an - an + b + a - b$$
$$= a \text{ (constant)}$$

Thus, difference between any two consecutive terms of the given progression is constant hence the given progression is an A.P.

S7. Given, $a_n = n(n + 2)$

For $n = 1$, $a_1 = 1(1 + 2) = 3$

For $n = 2$, $a_2 = 2(2 + 2) = 8$

For $n = 3$, $a_3 = 3(3 + 2) = 15$

For $n = 4$, $a_4 = 4(4 + 2) = 24$

For $n = 5$, $a_5 = 5(5 + 2) = 35$

Hence, first five terms are 3, 8, 15, 24, 35.

S8. Let a be the first term and d be the common difference of given A.P.

Here, $d = t_n - t_{n-1}$

$= [S_n - S_{n-1}] - [S_{n-1} - S_{n-2}]$

$= S_n - 2S_{n-1} + S_{n-2}$.

S9. Let a be the first term and d be the common difference of given A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{101} = 1212$$

$\Rightarrow$ $\frac{101}{2}[2a + (101 - 1)d] = 1212$

$\Rightarrow$ $a + 50d = 12$

Hence middle terms is $t_{51} = 12$.

S10. Let A.P. be $a - 3d$, $a - d$, $a + d$, $a + 3d$

$a - 3d + a + 3d = 8 \Rightarrow 2a = 8 \Rightarrow a = 4$

$\because$ $(a - d)(a + d) = 15 \Rightarrow a^2 - d^2 = 15$

$\Rightarrow$ $-d^2 = 15 - a^2 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

$\therefore$ Hence A.P be 1, 3, 5,7 (when d = 1) or 7, 5, 3, 1 when (d = -1). So least no of A.P is 1

S11. Since a, q, c are in A.P.

$\therefore$ $2q = a + c$ $\left\{\because\ 2q = \frac{1}{p} + \frac{1}{r};\ a + c = \frac{2}{b}\right\}$

$\Rightarrow$ $\frac{1}{p} + \frac{1}{r} = \frac{2}{b}$

$\Rightarrow$ $\frac{1}{p}, \frac{1}{b}, \frac{1}{r}$ are in A.P.

S12. Given that a, b, c are in A.P.

$\therefore$ $2b = a + c$

$\Rightarrow$ $8b^3 = (a + c)^3$

$\Rightarrow \qquad 8b^3 = a^3 + c^3 + 3ac\,(2b)$

$\Rightarrow \qquad a^3 + c^3 - 8b^3 = -6abc.$

S13. a, b, c are in A.P.

$\Rightarrow \quad \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.(Dividing by abc)

$\Rightarrow \quad \left(a+\frac{1}{bc}\right), \left(b+\frac{1}{ca}\right), \left(c+\frac{1}{ab}\right)$ are also in A.P.

S14. Let a be the first term and d be the common difference of given A.P.

$$a = 14\frac{1}{2},\ d = \frac{-5}{2},\ a_n = -38$$

$\Rightarrow \qquad -38 = \frac{29}{2} + (n-1)\left(\frac{-5}{2}\right)$

$\Rightarrow \qquad \frac{-5}{2}(n-1) = -\frac{105}{2} \qquad \Rightarrow \qquad n = 21.$

Hence, there are 21 terms in this A.P.

S15. $\qquad t_5 = 13, \quad t_{10} = 28$

$\Rightarrow \qquad a + 4d = 13$

and $\qquad a + 9d = 28$

Solving these equations, we get

$$a = 1,\ d = 3$$

Therefore, the A.P. is 1, 4, 7, ...

$$t_{20} = a + 19d = 1 + 19(3) = 58$$

S16. Given, $\qquad a_n = 2^n$

For $n = 1$, $\qquad a_1 = 2^1 = 2$

For $n = 2$, $\qquad a_2 = 2^2 = 4$

For $n = 3$, $\qquad a_3 = 2^3 = 8$

For $n = 4$, $\qquad a_4 = 2^4 = 16$

For $n = 5$, $\qquad a_5 = 2^5 = 32$

Hence, first five terms of the given sequence are 2, 4, 8, 16 and 32.

S17. Given, $\qquad a_n = \frac{n}{n+1}$

For $n = 1$, $\qquad a_1 = \frac{1}{1+1} = \frac{1}{2}$

For $n = 2$, $a_2 = \frac{2}{2+1} = \frac{2}{3}$

For $n = 3$, $a_3 = \frac{3}{3+1} = \frac{3}{4}$

For $n = 4$, $a_4 = \frac{4}{4+1} = \frac{4}{5}$

For $n = 5$, $a_5 = \frac{5}{5+1} = \frac{5}{6}$

The first 5 terms of the given sequence are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$.

S18. Given, $a_n = 4n - 3$

For $n = 17$, $a_{17} = 4 \times 17 - 3 = 68 - 3 = 65$

For $n = 24$, $a_{24} = 4 \times 24 - 3 = 96 - 3 = 93$

Hence, $a_{17} = 65$ and $a_{24} = 93$.

S19. Given, $a_n = \frac{n(n^2+5)}{4}$

For $n = 1$, $a_1 = \frac{1(1^2+5)}{4} = \frac{6}{4} = \frac{3}{2}$

For $n = 2$, $a_2 = \frac{2(2^2+5)}{4} = \frac{2 \times 9}{4} = \frac{9}{2}$

For $n = 3$, $a_3 = \frac{3(3^2+5)}{4} = \frac{3 \times 14}{4} = \frac{21}{2}$

For $n = 4$, $a_4 = \frac{4(4^2+5)}{4} = \frac{4 \times 21}{4} = 21$

For $n = 5$, $a_5 = \frac{5(5^2+5)}{4} = \frac{5 \times 30}{4} = \frac{75}{2}$

Hence, the first 5 terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}$, 21 and $\frac{75}{2}$.

S20. Given, $a_n = (-1)^{n-1} \cdot 5^{n+1}$

For $n = 1$, $a_1 = (-1)^{1-1} \cdot 5^{1+1} = 25$

For $n = 2$, $a_2 = (-1)^{2-1} \cdot 5^{2+1} = -125$

For $n = 3$, $a_3 = (-1)^{3-1} \cdot 5^{3+1} = 625$

For $n = 4$, $a_4 = (-1)^{4-1} \cdot 5^{4+1} = -3125$

For $n = 5$, $a_5 = (-1)^{5-1} \cdot 5^{5+1} = 15625$

Hence, first 5 terms of the given sequence are 25, – 125, 625, – 3125, 15625.

S21. Given, $a_n = \dfrac{2n-3}{6}$

For $n = 1$, $a_1 = \dfrac{2\times1-3}{6} = \dfrac{2-3}{6} = -\dfrac{1}{6}$

For $n = 2$, $a_2 = \dfrac{2\times2-3}{6} = \dfrac{4-3}{6} = \dfrac{1}{6}$

For $n = 3$, $a_3 = \dfrac{2\times3-3}{6} = \dfrac{3}{6} = \dfrac{1}{2}$

For $n = 4$, $a_4 = \dfrac{2\times4-3}{6} = \dfrac{5}{6}$

For $n = 5$, $a_5 = \dfrac{2\times5-3}{6} = \dfrac{7}{6}$

Hence, the first 5 terms are $-\dfrac{1}{6}, \dfrac{1}{6}, \dfrac{1}{2}, \dfrac{5}{6}$ and $\dfrac{7}{6}$.

S22. Given $a_n = \dfrac{n(n-2)}{n+3}$

For $n = 20$, $a_{20} = 20\times\dfrac{(20-2)}{20+3} = \dfrac{20\times18}{23} = \dfrac{360}{23}$

Hence, $a_{20} = \dfrac{360}{23}$.

S23. Given, $a_n = (-1)^{n-1}\, n^3$

For $n = 9$, $a_9 = (-1)^{9-1}\cdot(9)^3$

$= 9\times9\times9 = 729$

Hence, $a_9 = 729$.

S24. Given $a_7 = \dfrac{n^2}{2^n}$

For $n = 7$, $a_7 = \dfrac{(7)^2}{2^7} = \dfrac{49}{128}$

Hence, $a_7 = \dfrac{49}{128}$

S25. Let a be the first term and d be the common difference of given A.P.

We have, $a_m = a + (m-1)\,d = n,$ ... (i)

and $a_n = a + (n-1)\,d = m$ ... (ii)

Solving Eq. (i) and (ii), we get

$(m-n)\,d = n - m$ or $d = -1$

$\Rightarrow$ $a = n + m - 1$

Therefore, $a_p = a + (p-1)d$

$= n + m - 1 + (p-1)(-1)$

$= n + m - p$

Hence, the p^{th} term is $n + m - p$.

S26. Putting $n = 20$, we obtain

$$a_{20} = (20-1)(2-20)(3+20)$$
$$= 19 \times (-18) \times (23) = -7866.$$

S27. It is given that

$$11t_{11} = 7t_7$$

$\Rightarrow$ $11[a + 10d] = 7[a + 6d]$

$\Rightarrow$ $4a + 68d = 0$

$\Rightarrow$ $a + 17d = 0$

$\Rightarrow$ $t_{18} = 0$ Proved

Note: If m times m^{th} term of an A.P. is equal to the n times the n^{th} term then $(m + n)^{th}$ term of the A.P. is zero.

S28. Here, $T_n = 2n + 3$

$T_1 = 2 \times 1 + 3 = 5$

$T_2 = 2 \times 2 + 3 = 7$

$T_3 = 2 \times 3 + 3 = 9$

$T_4 = 2 \times 4 + 3 = 11$

$\because$ $T_2 - T_1 = T_4 - T_3 = 2$ (constant)

Hence, it is an A.P. with common difference 2.

$\therefore$ $T_{10} = T_1 + 9 . d$

$= 5 + 9 \times 2 = 23.$

S29. Given, $T_n = (-1)^{n-1} \cdot \cos \frac{n\pi}{4}$

Putting $n = 1, 2, 3, 4, 5$, we get

$$T_1 = (-1)^{1-1} \cdot \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$T_2 = (-1)^{2-1} \cdot \cos \frac{2\pi}{4} = 0$$

$$T_3 = (-1)^{3-1} \cdot \cos \frac{3\pi}{4} = \frac{-1}{\sqrt{2}}$$

$$T_4 = (-1)^{4-1} \cdot \cos \frac{4\pi}{4} = +1$$

$$T_5 = (-1)^{5-1} \cdot \cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}.$$

S30. $\because$ $\sin n\pi = 0$ $(n \in I)$

$$T_1 = (-1)^1 \sin\frac{\pi}{2} = -1$$

$$T_2 = (-1)^2 \sin \pi = 0$$

$$T_3 = (-1)^3 \sin\frac{3\pi}{2} = 1$$

$$T_4 = (-1)^4 \sin 2\pi = 0$$

$$T_5 = (-1)^5 \sin\frac{5\pi}{2} = -1.$$

S31. Given, $a_n = a_{n-1} + 3$

Hence, $a_2 = a_1 + 3 = 1 + 3 = 4$

$a_3 = a_2 + 3 = 4 + 3 = 7$

$a_4 = a_3 + 3 = 7 + 3 = 10$

$a_5 = a_4 + 3 = 10 + 3 = 13$

$a_6 = a_5 + 3 = 13 + 3 = 16$

S32. (a) Here, $a = 5, \quad d = 3, \quad t_n = 320$

$t_n = a + (n-1)d$

$\Rightarrow$ $320 = 5 + (n-1) \cdot 3$

$\Rightarrow$ $n = 106$

Therefore, 106th term will be 320.

(b) Here, $a = 4, \quad d = 5, \quad t_n = 114$

$t_n = a + (n-1)d$

$\Rightarrow$ $114 = 4 + (n-1)5$

$\Rightarrow$ $n = 23$

Therefore, 23rd term will be 114.

S33. Given, $T_n = \dfrac{T_{n-1}}{n+1}$

Putting $n = 2, 3, 4, 5, 6$, we get

$$T_2 = \frac{T_1}{3} = \frac{-5}{3},$$

$$T_3 = \frac{T_2}{4} = \frac{-5}{12} = \frac{-5}{12}$$

$$T_4 = \frac{T_3}{5} = \frac{-5}{12} \times \frac{1}{5} = \frac{-1}{12}$$

$$T_5 = \frac{T_4}{6} = \frac{-1}{12} \times \frac{1}{6} = \frac{-1}{72}$$

$$T_6 = \frac{T_5}{7} = \frac{-1}{72} \times \frac{1}{7} = \frac{-1}{504}.$$

S34. If t_r be the r^{th} term of the A.P. then

$$t_r = S_r - S_{r-1}$$

$$= cr(r-1) - c(r-1)(r-2)$$

$$= c(r-1)(r-r+2) = 2c(r-1)$$

$\because$ $$t_1^2 + t_2^2 + \ldots + t_n^2 = 4c^2(0^2 + 1^2 + 2^2 + \ldots + (n-1)^2$$

$$= \frac{4c^2(n-1)(n)(2n-1)}{6}$$

$$= \frac{2}{3}c^2 n(n-1)(2n-1)$$

S35. Let a be the first term and d be the common difference of given A.P.

$$a_n = a + (n-1)d$$

$\therefore$ n^{th} term of the series is $20 + (n-1)\left(\frac{-2}{3}\right)$

For the sum to be maximum

$$n^{\text{th}} \text{ term} \geq 0$$

$\Rightarrow$ $$20 + (n-1)\left(\frac{-2}{3}\right) \geq 0$$

$\Rightarrow$ $$n \leq 31$$

Thus the sum of 31 terms is maximum and is

$$= \frac{31}{2}\left[2 \times 20 + (31-1)\left(\frac{-2}{3}\right)\right] \qquad \left\{S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$

$$= \frac{31}{2}\left[40 + 30 \times \left(\frac{-2}{3}\right)\right] = 310.$$

S36. $\because$ $t_4 = 3t_1, \quad t_7 = 2t_3 + 1$

$\Rightarrow$ $a + 3d = 3a, \quad a + 6d = 2(a + 2d) + 1$

$\Rightarrow$ $2a - 3d = 0$ ... (i)

$a - 2d = -1$ ... (ii)

Solving these Eq., we get

$$a = 3$$

$$d = 2$$

Hence first term is 3 and common difference is 2.

S37. Let a_1 be the first term and d be the common difference of given A.P.

$$S_n = \frac{n}{2}[a_1 + a_n]$$

Given $S_n = 2550$

Hence, $\frac{50}{2}[a_1 + a_{99}] = 2550$

$\Rightarrow$ $a_1 + a_{99} = 102$

Now, the sum of all the terms is

$$\frac{99}{2}[a_1 + a_{99}] = \frac{99}{2} \cdot 102 = 5049.$$

S38. Let a be the first term and d be the common difference of given A.P

$\because$ $T_m = \frac{1}{n} \quad \Rightarrow \quad a + (m-1) \cdot d = \frac{1}{n}$ ... (i)

and $T_n = \frac{1}{m} \quad \Rightarrow \quad a + (n-1) \cdot d = \frac{1}{m}$ ... (ii)

Subtracting Eq. (ii) from Eq. (i), we get

$$(m - n) \cdot d = \left(\frac{1}{n} - \frac{1}{m}\right) \quad \Rightarrow \quad d = \frac{1}{mn}$$

$\therefore$ $a = \left[\frac{1}{n} - \left(\frac{m-1}{mn}\right)\right] = \frac{1}{mn}$

$\therefore$ $T_{mn} = a + (mn - 1) \cdot d = \frac{1}{mn} + (mn - 1) \cdot \frac{1}{mn} = 1$

S39. Given, first term $a = 7 - 4i$ and $d = -1 + 2i$

$$a_n = (7 - 4i) + (n - 1)(-1 + 2i)$$

$= (7 - n + 1) + i(-4 + 2n - 2)$

$= (8 - n) + (2n - 6)i$ $\{a_n = a + (n-1)d\}$

(i) For purely real $2n - 6 = 0 \Rightarrow n = 3$ *i.e.*, 3^{rd} term.

(ii) For purely imaginary $8 - n = 0 \Rightarrow n = 8^{th}$ term

S40. Here, $a = 121, \quad d = -4$

Let t_n be first negative term.

$t_n = a + (n-1)d$

$= 121 + (n-1)(-4)$

$= 125 - 4n$

$t_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow n > \frac{125}{4}$

$\Rightarrow n > 31.25$

$\Rightarrow n = 32$

$\Rightarrow$ 32^{nd} will be the first negative term.

S41. $t_9 = 0$

$\Rightarrow a + 8d = 0$

$\Rightarrow a = -8d$

$t_{29} = a + 28d$

$= -8d + 28d = 20d$ ... (i)

$t_{19} = a + 18d$

$= -8d + 18d = 10d$ ... (ii)

From (i) and (ii), we get

$t_{29} = 2t_{19}$

S42. Let a be the first term and d be the common difference of given A.P.

Given, $t_6 = 12$,

$\Rightarrow a + 5d = 12$ ... (i)

$t_8 = 22$

Similarly, $a + 7d = 22$ ... (ii)

Solving Eq. (i) and (ii), we get

$$a = -13,\ d = 5$$

$$t_2 = a + d = -13 + 5 = -8$$

$$t_r = a + (r-1)d = -13 + (r-1)\cdot 5 = 5r - 18$$

S43. (a) Here, $a = 10, \quad d = 3, \quad t_n = 49$

$$t_n = a + (n-1)d$$

$\Rightarrow$ $49 = 10 + (n-1)\cdot 3$

$\Rightarrow$ $n = 14$

(b) Here, $a = 7, \quad d = 6, \quad t_n = 199$

$$t_n = a + (n-1)d$$

$\Rightarrow$ $199 = 7 + (n-1)\cdot 6$

$\Rightarrow$ $n = 33$

S44. (a) Since, $2K + 1, 2K - 1, 3K + 4$ are in A.P.

$\Rightarrow$ $2(2K - 1) = (2K + 1) + (3K + 4)$

$\Rightarrow$ $4K - 2 = 5K + 5$

$\Rightarrow$ $K = -7.$

(b) Since, $3K - 1, K + 1, K + 3$ are in A.P.

$\Rightarrow$ $2(K + 1) = (3K - 1) + (K + 3)$

$\Rightarrow$ $2K + 2 = 4K + 2$

$\Rightarrow$ $K = 0.$

S45. Let a be the first term and d be the common difference of given A.P.

$$m \cdot t_m = nT_n$$

$\Rightarrow$ $m[a + (m-1)d] = n[a + (n-1)d]$

$\Rightarrow$ $[(m^2 - n^2) - (m-n)]d = (n-m)a$

$\Rightarrow$ $[(m-n)(m+n) - (m-n)]d = (n-m)a$

$\Rightarrow$ $(m-n)(m+n-1)d = (n-m)a$

$\Rightarrow$ $(m+n-1)d = -a \quad \Rightarrow \quad a + (m+n-1)d = 0$

$\Rightarrow$ $T_{m+n} = 0$

S46. Given,

$$a_1 = -1, \quad a_n = \frac{a_{n-1}}{n}$$

$$a_2 = \frac{a_1}{2} \quad = \quad a_2 = \frac{-1}{2}$$

$$a_3 = \frac{a_2}{3} \quad = \frac{-1}{2} \times \frac{1}{3} = \frac{-1}{6}$$

$$a_4 = \frac{a_3}{4} \quad = \frac{-1}{4} \times \frac{1}{6} = \frac{-1}{24}$$

$$a_5 = \frac{a_4}{5} \quad = \frac{-1}{24} \times \frac{1}{5} = \frac{-1}{120}$$

$$a_6 = \frac{a_5}{6} \quad = \frac{-1}{120} \times \frac{1}{6} = \frac{-1}{720}.$$

S47. Let a be the first term and d be the common difference of given A.P.

$\because$ First and last numbers satisfying the given condition are 11 and 96 respectively, therefore the numbers are: 11, 16, 21, ... 96.

Here, $a = 11$, $d = 5$, $t_n = 96$

$$t_n = a + (n-1)d$$

$\Rightarrow$ $$96 = 11 + (n-1)5 \quad \Rightarrow \quad n = 18$$

Hence there are 18 two digit numbers which leaves the remainder 1 when divided by 5.

S48. Let a be the first term and d be the common difference of given A.P.

Let the three number be $(a - d)$, a, $(a + d)$.

$\because$ $$(a - d) + a + (a + d) = 27$$

$\Rightarrow$ $$3a = 27 \quad \Rightarrow \quad a = 9$$

$$(a - d)(a + d) = 77$$

$\Rightarrow$ $$a^2 - d^2 = 77 \quad \Rightarrow \quad d^2 = 81 - 77$$

$\Rightarrow$ $$d^2 = 4 \quad \Rightarrow \quad d = \pm 2$$

Hence, the numbers are 7, 9, 11 or 11, 9, 7.

S49. If $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P., then

$$\frac{2}{ca} = \frac{1}{bc} + \frac{1}{ab} \quad \text{... (i)}$$ Multiplying Eq. (i), by abc we get

$$\frac{2abc}{ca} = \frac{abc}{bc} + \frac{abc}{ab}$$

$\Rightarrow$ $$2b = a + c$$

$\Rightarrow$ a, b, c are in A.P.. Hence the problem.

S50. Let a be the first term and d be the common difference of given A.P.

$\because$ $$t_n = a + (n-1)d$$

$$T_p + T_{p+2q} = a + (p-1)d + a + (p + 2q - 1)d$$
$$= 2a + 2(p + q - 1)d = 2[a + (p + q - 1)d]$$
$$= 2T_{p+q}.$$

S51. Let a be the first term and d be the common difference of given A.P.

Here, $n = 60$, $a = 7$, $t_n = 125$

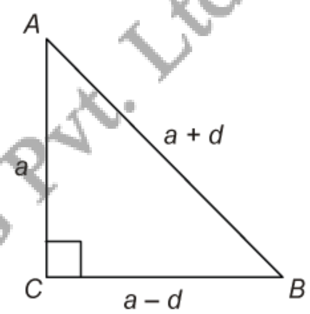
Now, $t_n = a + (n-1)d$

$\Rightarrow$ $125 = 7 + (60 - 1)d$

$\Rightarrow$ $d = 2$

$$t_{25} = a + 24d$$
$$= 7 + 24(2) = 55.$$

S52. Let $\angle C = 90°$, $\angle B = (90 - A)$



The sides are $(a - d)$, a, $(a + d)$

We have,

$$(a + d)^2 = (a - d)^2 + a^2$$

(using pythagoras theorem)

$\Rightarrow$ $4ad - a^2 = 0 \Rightarrow a = 4d$

Hence the sides are $3d$, $4d$, $5d$

$\therefore$ $$\sin A = \frac{BC}{AB} = \frac{a-d}{a+d} = \frac{3d}{5d} = \frac{3}{5}$$

$$\sin B = \frac{AC}{AB} = \frac{a}{a+d} = \frac{4d}{5d} = \frac{4}{5}.$$

S53. We know that in an A.P., the sum of terms equidistant from the beginning and end is always same and is equal to the sum of first and last term.

So, if an A.P., consists of 24 terms then

$$a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$$

$\Rightarrow$ $(a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow$ $3(a_1 + a_{24}) = 225 \Rightarrow a_1 + a_{24} = \frac{225}{3} = 75$

$\therefore$ $$S_{24} = \frac{24}{2}(a_1 + a_{24})$$
$$= 12 \times 75 = 900.$$

S54. S_1 is an A.P. of n terms, but S_2 is an A.P. of $\frac{n-1}{2}$ terms with common difference $2d$.

$$S_1 = \frac{n}{2}[T_1 + T_n]$$

$$S_2 = \frac{1}{2}\left(\frac{n-1}{2}\right)[T_2 + T_{n-1}]$$

$$= \frac{1}{2}\left(\frac{n-1}{2}\right)[T_1 + T_n]$$

$$\therefore \qquad \frac{S_1}{S_2} = \frac{2n}{n-1}.$$

S55. Here, $a = 100$, $t_n = 1000$, $S_n = 5500$

We know that

$$S_n = \frac{n}{2}[a + t_n]$$

$$\Rightarrow \qquad 5500 = \frac{n}{2}[100 + 1000] \quad \Rightarrow \quad n = 10.$$

S56. Since, $t_n = 3n + 4$, $a = t_1 = 3 \times 1 + 4 = 7$

$$S_n = \frac{n}{2}[a + t_n] = \frac{n}{2}[7 + (3n + 4)]$$

$$= \frac{n(3n+11)}{2}.$$

S57.

$$S_n = 2n^2 + 3n$$

$$S_{n-1} = 2(n-1)^2 + 3(n-1)$$

$$= 2(n^2 - 2n + 1) + 3(n-1) = 2n^2 - n - 1$$

We know that $t_n = S_n - S_{n-1} = (2n^2 + 3n) - (2n^2 - n - 1) = 4n + 1$

As, n^{th} term is linear in n, we will get an A.P.

$$t_r = 4r + 1.$$

S58. Let $S_n = pn^2 + qn + r$

$$\Rightarrow \qquad S_{n-1} = p(n-1)^2 + q(n-1) + r$$

$$= p(n^2 - 2n + 1) + q(n-1) + r$$

$$= pn^2 + (q - 2p)n + p - q + r$$

We know that $t_n = S_n - S_{n-1}$

$$= [pn^2 + qn + r] - [pn^2 + (q-2p)n + p - q + r]$$

$$= 2pn + q - p$$

As, n^{th} term is linear expression in n, we will get an A.P.

S59. Let a be the first term and d be the common difference of the given A.P. then

$$S_2 - S_1 = \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$$

$$= na + \frac{n(3n-1)}{2}\cdot d = \frac{n}{2}[2a + (3n-1)d]$$

$$\therefore \quad 3(S_2 - S_1) = \frac{3n}{2}\{2a + (3n-1)d\} = S_3.$$

S60. (a) $\because \quad T_n = (-1)^{n-1}\cdot 5^{n+1}$ ... (i)

Putting $n = 1, 2, 3, 4, 5$ in equation (i), we get

$$T_1 = (-1)^{1-1}\cdot(5)^{1+1} = 25$$

$$T_2 = (-1)^{2-1}\cdot(5)^{2+1} = -125$$

$$T_3 = (-1)^{3-1}\cdot(5)^{3+1} = 625$$

$$T_4 = (-1)^{4-1}\cdot(5)^{4+1} = -3125$$

$$T_5 = (-1)^{5-1}\cdot(5)^{5+1} = 15625$$

(b) $T_n = \frac{n(n^2+5)}{4}$ ... (ii)

Putting $n = 1, 2, 3, 4, 5$ in Eq. (ii), we get

$$T_1 = \frac{1(1^2+5)}{4} = \frac{6}{4} = \frac{3}{2}$$

$$T_2 = \frac{2(2^2+5)}{4} = \frac{18}{4} = \frac{9}{2}$$

$$T_3 = \frac{3(3^2+5)}{4} = \frac{42}{4} = \frac{21}{2}$$

$$T_4 = \frac{4(4^2+5)}{4} = \frac{84}{4} = 21$$

$$T_5 = \frac{5(5^2+5)}{4} = \frac{150}{4} = \frac{75}{2}.$$

S61.

Ratio of sum of n terms $= \frac{3n+8}{7n+15}$

$$\frac{S_n}{S'_n} = \frac{3n+8}{7n+15}$$

$$\frac{T_1 + T_n}{T'_1 + T'_n} = \frac{3n+8}{7n+15}$$

$$\frac{a_1 + a_1 + (n-1)d}{a'_1 + a'_1 + (n-1)d'} = \frac{3n+8}{7n+15}$$

Replace $n \to (2n-1)$

$$\frac{a_1 + a_1 + 2(n-1)d}{a_1' + a_1' + 2(n-1)d'} = \frac{3(2n-1)+8}{7(2n-1)+15}$$

$$\frac{a_1 + (n-1)d}{a_1' + (n-1)d'} = \frac{3(2n-1)+8}{7(2n-1)+15}$$

$$\frac{T_n}{T_n'} = \frac{6n+5}{14n+8}$$

$\therefore$ Ratio of 12th terms $= \dfrac{6\times 12+5}{14\times 12+8} = \dfrac{77}{176} = \dfrac{7}{16}$.

S62. (a) $\because$ $a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1$

$a_3 = a_{3-1} - 1 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_{4-1} - 1 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_{5-1} - 1 = a_4 - 1 = 0 - 1 = -1$

$a_6 = a_{6-1} - 1 = a_5 - 1 = -1 - 1 = -2$

$a_7 = a_{7-1} - 1 = a_6 - 1 = -2 - 1 = -3$

(b) We have $a_1 = a_2 = 1, \quad a_n = a_{n-1} + a_{n-2}$

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

$a_6 = a_5 + a_4 = 5 + 3 = 8$

$a_7 = a_6 + a_5 = 8 + 5 = 13$

For $n = 1, \quad \dfrac{a_{n+1}}{a_n} = \dfrac{a_2}{a_1} = \dfrac{1}{1} = 1$

$n = 2, \quad \dfrac{a_{n+1}}{a_n} = \dfrac{a_3}{a_2} = \dfrac{2}{1} = 2$

$n = 3, \quad \dfrac{a_{n+1}}{a_n} = \dfrac{a_4}{a_3} = \dfrac{3}{2}$

$n = 4, \quad \dfrac{a_{n+1}}{a_n} = \dfrac{a_5}{a_4} = \dfrac{5}{3}$

$n = 5, \quad \dfrac{a_{n+1}}{a_n} = \dfrac{a_6}{a_5} = \dfrac{8}{5}$.

S63. (a) Since, $T_n = 2n^2 - n + 1$

$T_1 = 2 \times 1^2 - 1 + 1 = 2$

$T_2 = 2 \times 2^2 - 2 + 1 = 7$

$T_3 = 2 \times 3^2 - 3 + 1 = 16$

$T_4 = 2 \times 4^2 - 4 + 1 = 29$

$T_5 = 2 \times 5^2 - 5 + 1 = 46$

(b) Since, $T_n = \frac{n^n}{3^n}$

$T_1 = \frac{1^1}{3^1} = \frac{1}{3}$

$T_2 = \frac{2^2}{3^2} = \frac{4}{9}$

$T_3 = \frac{3^3}{3^3} = 1$

$T_4 = \frac{4^4}{3^4} = \frac{256}{81}$

$T_5 = \frac{5^5}{3^5} = \frac{3125}{243}$.

S64. Since a, b, c are in A.P.

$\Rightarrow$ $2b = a + c$

From R.H.S.

$$3b(a^2 + c^2) = \frac{3}{2}(a + c)(a^2 + c^2)$$

$$= \frac{3}{2}(a^3 + c^3 + ac^2 + a^2c)$$

From L.H.S.

$$a^3 + 4b^3 + c^3 = a^3 + 4\left(\frac{a+c}{2}\right)^3 + c^3$$

$$= \frac{3}{2}(a^3 + c^3 + ac^2 + a^2c)$$

$\therefore$ L.H.S. = R.H.S.

S65. $\because$ $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

$\therefore$ $b^2(c + a) - a^2(b + c) = c^2(a + b) - b^2(c + a)$

$\Rightarrow$ $c(b^2 - a^2) + ab(b - a) = a(c^2 - b^2) + bc(c - b)$

$\Rightarrow \quad (b - a)(ab + bc + ca) = (c - b)(ab + bc + ac)$

$\Rightarrow \quad (ab + bc + ac)[2b - (a + c)] = 0$

$\Rightarrow \quad$ Either $2b = (a + c)$ or $ab + bc + ac = 0$

$\Rightarrow$ Either a, b, c are in A.P. or $ab + bc + ac = 0$

S66. Let d be the common difference of the given A.P. then,

$$a_2 - a_1 = a_3 - a_2 = \ldots\ldots = a_n - a_{n-1} = d$$

From L.H.S.

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + \ldots + \frac{1}{a_{n-1} a_n} = \frac{1}{d}\left[\frac{d}{a_1 a_2} + \frac{d}{a_2 a_3} + \frac{d}{a_3 a_4} + \ldots + \frac{d}{a_{n-1} a_n}\right]$$

$$= \frac{1}{d}\left[\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + \frac{a_4 - a_3}{a_3 a_4} + \ldots\ldots + \frac{a_n - a_{n-1}}{a_{n-1} a_n}\right]$$

$$= \frac{1}{d}\left[\left(\frac{1}{a_1} - \frac{1}{a_2}\right) + \left(\frac{1}{a_2} - \frac{1}{a_3}\right) + \left(\frac{1}{a_3} - \frac{1}{a_4}\right) + \ldots + \left(\frac{1}{a_{n-1}} - \frac{1}{a_n}\right)\right]$$

$$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_n}\right] = \frac{1}{d}\left[\frac{a_n - a_1}{a_1 a_n}\right]$$

$$= \frac{1}{d}\left[\frac{a_1 + (n-1)d - a_1}{a_1 a_n}\right] = \frac{n-1}{a_1 a_n} \textbf{ R.H.S.}$$

S67. Let a be the first term and d be the common difference of the A.P.

$\therefore \quad k^{th}$ term from beginning $= T_{k_1} = a + (k-1) \cdot d$ ... (i)

Let l be the last term of the A.P. and $l = a + (n-1) \cdot d$

$\therefore$ Thus the required A.P. is $a, a + d, a + 2d, \ldots, l - 2d, l - d, l$.

The A.P. in reverse order can be written as

$$= l, l - d, \ldots a + 2d, a + d, a$$

$\therefore \quad k^{th}$ term from the end of the given A.P.

$$T_{k_2} = [l + (k-1)(-d)] = [a + (n-1)d - (k-1)d]$$

$$= a + (n - 1 - k + 1)d = a + (n-k)d \quad \ldots \text{(ii)}$$

Adding (i) and (ii), we get

$$\text{Required sum} = T_{k_1} + T_{k_2} = [a + (k-1)d] + [a + (n-k)d]$$

$$= 2a + [k - 1 + n - k]d$$

$$= 2a + (n-1)d$$

Which is independent of k.

Also sum of first and last terms $= a + l = a + [a + (n - 1)d] = 2a + (n - 1)d$

Hence, the sum of first and last terms is independent of k.

S68. (a) Since,

$$T_n = \frac{2n - 3}{4}$$

$$T_1 = \frac{2 \times 1 - 3}{4} = -\frac{1}{4}$$

$$T_2 = \frac{2 \times 2 - 3}{4} = \frac{1}{4}$$

$$T_3 = \frac{2 \times 3 - 3}{4} = \frac{3}{4}$$

$$T_4 = \frac{2 \times 4 - 3}{4} = \frac{5}{4}.$$

(b) Since,

$$T_n = (-1)^{n-1} \cdot 2^{n+1}$$

$$T_1 = (-1)^{1-1} \cdot 2^{1+1} = 2^2 = 4$$

$$T_2 = (-1)^{2-1} \cdot 2^{2+1} = -8$$

$$T_3 = (-1)^{3-1} \cdot 2^{3+1} = 16$$

$$T_4 = (-1)^{4-1} \cdot 2^{4+1} = -32.$$

S69. (a) $\because$

$$T_n = \frac{(-1)^{n-1}}{n^3}$$

$$T_1 = \frac{(-1)^{1-1}}{1^3} = 1$$

$$T_2 = \frac{(-1)^{2-1}}{2^3} = -\frac{1}{8}$$

$$T_3 = \frac{(-1)^{3-1}}{3^3} = \frac{1}{27}$$

$$T_4 = \frac{(-1)^{4-1}}{4^3} = -\frac{1}{64}$$

$$T_5 = \frac{(-1)^{5-1}}{5^3} = \frac{1}{125}$$

(b) $\because$

$$T_n = \left(\frac{1}{2}\right)^{2n+1} + \left(-\frac{1}{2}\right)^{2n}$$

$$T_1 = \left(\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

$$T_2 = \left(\frac{1}{2}\right)^5 + \left(-\frac{1}{2}\right)^4 = \frac{1}{32} + \frac{1}{16} = \frac{3}{32}$$

$$T_3 = \left(\frac{1}{2}\right)^7 + \left(-\frac{1}{2}\right)^6 = \frac{1}{64} + \frac{1}{128} = \frac{3}{128}$$

$$T_4 = \left(\frac{1}{2}\right)^9 + \left(-\frac{1}{2}\right)^8 = \frac{1}{512} + \frac{1}{256} = \frac{3}{512}$$

$$T_5 = \left(\frac{1}{2}\right)^{11} + \left(-\frac{1}{2}\right)^{10} = \frac{1}{1024} + \frac{1}{2048} = \frac{3}{2048}.$$

S70. $t_p = a, \quad t_q = b, \quad t_r = c \quad \Rightarrow \quad A + (p-1)D = a$

$$A + (q-1)D = b$$

$$A + (r-1)D = c$$

Now,

$$a(q-r) + b(r-p) + c(p-q) = [A + (p-1)D]\,(q-r) + [A + (q-1)D]\,(r-p) + [A + (r-1)D]\,(p-q)$$

$$= A[q - r + r - p + p - q] + D[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= A(0) + D(0) = 0.$$

S71. Here, $S_n = n^2p$ and $S_m = m^2p$

$\Rightarrow \quad \frac{n}{2}[a + t_n] = n^2p$

and $\quad \frac{m}{2}[a + t_m] = m^2p$

$\Rightarrow \quad a + t_n = 2np \quad$ ... (i)

$\Rightarrow \quad a + t_m = 2mp \quad$ ... (ii)

Subtracting, we get

$$t_n - t_m = 2(n-m)p$$

$\Rightarrow \quad [a + (n-1)d] - [a + (m-1)d] = 2(n-m)p$

$\Rightarrow \quad (n-m)d = 2(n-m)p$

$\Rightarrow \quad d = 2p \quad$ ... (iii)

From Eq. (i), we get

$$a + a + (n-1)d = 2np$$

$\Rightarrow \quad 2a + (n-1)d = 2np$

$2a + (n-1)\cdot 2p = 2np \quad$ ... [From Eq. (iii)]

$$2a = 2p \quad \Rightarrow \quad a = p$$

Now, $$S_p = \frac{p}{2}[2a + (p-1)d]$$

$$= \frac{p}{2}[2p + (p-1)\cdot 2p] = p^3.$$

S72. Let a be the first term and d be the common difference of given A.P.

We have to prove that

$$t_{n+1} + t_{n+2} + \ldots + t_{2n} = \frac{1}{3}[t_1 + t_2 + \ldots + t_{3n}]$$

Now,

$$t_{n+1} + t_{n+2} + \ldots + t_{2n} = \frac{n}{2}[t_{n+1} + t_{2n}]$$

$$= \frac{n}{2}[a + nd + a + (2n-1)d]$$

$$= \frac{1}{3}\cdot\frac{3n}{2}[2a + (3n-1)d]$$

$$= \frac{1}{3}\cdot[t_1 + t_2 + \ldots + t_{3n}]$$

S73. Let a be the first term and d be the common difference of given A.P.

Here, $S_p = q, \quad S_q = p$

$\Rightarrow$ $$\frac{p}{2}[2a + (p-1)d] = q$$

and $$\frac{q}{2}[2a + (q-1)d] = p$$

$\Rightarrow$ $$p[(2a-d) + pd] = 2q$$

and $q[(2a-d) + qd] = 2p$ Subtracting, we get

$$(p-q)(2a-d) + (p^2-q^2)d = 2(q-p)$$

$\Rightarrow$ $$(p-q)(2a-d) + (p-q)(p+q)d = -2(p-q)$$

$\Rightarrow$ $$2a - d + (p+q)d = -2 \quad \ldots \text{(i)}$$

Now, $$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$$= \frac{p+q}{2}(-2) \quad \ldots \text{[From (i)]}$$

$$= -(p+q).$$

S74. Let a be the first term and d be the common difference of given A.P.

$S_p = 0 \quad \Rightarrow \frac{p}{2}[2a + (p-1)d] = 0$

$\Rightarrow \qquad 2a + (p-1)d = 0$

$\Rightarrow \qquad d = -\frac{2a}{p-1} \qquad \text{... (i)}$

As sum of first p terms is 0, the sum of next q terms is same as the sum of first $p + q$ terms.

$$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$$= \frac{p+q}{2}\left[2a + (p+q-1)\left(\frac{-2a}{p-1}\right)\right] \qquad \text{[From Eq. (i)]}$$

$$= -\frac{aq(p+q)}{p-1}.$$

S75. Here, $\qquad S_p = S_q$

$\Rightarrow \qquad \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$\Rightarrow \qquad 2a(p-q) + [p(p-1) - q(q-1)]d = 0$

$\Rightarrow \qquad 2a(p-q) + [p^2 - q^2] - (p-q)]d = 0$

$\Rightarrow \qquad 2a(p-q) + (p-q)[p+q-1]d = 0$

$\Rightarrow \qquad 2a + (p+q-1)d = 0 \qquad \text{... (i)}$

Now, $\qquad S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$$= \frac{p+q}{2}(0) \qquad \text{[From Eq. (i)]}$$

$$= 0.$$

S76. Let the four parts be $a - 3d, a - d, a + d, a + 3d$ then sum of four parts = 32

$\Rightarrow \quad (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow \qquad 4a = 32 \quad \Rightarrow \quad a = 8$

and $\qquad \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \qquad \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15} \quad \Rightarrow \quad \frac{64 - 9d^2}{64 - d^2} = \frac{7}{15}$

$\Rightarrow \qquad d^2 = 4 \quad \Rightarrow \quad d = \pm 2$

Hence the required parts are 2, 6, 10, 14 or 14,10, 6, 2

S77. $\because \quad a = 20, \quad d = -4$, and n^{th} term $= -176$

$$T_n = a + (n-1)\cdot d$$

$$\Rightarrow \quad -176 = 20 + (n-1)(-4)$$

$$\Rightarrow \quad (n-1) = \frac{-176-20}{-4} = \frac{196}{4}$$

$$\Rightarrow \quad (n-1) = 49 \quad \Rightarrow \quad n = 50$$

Hence the middle terms are 25^{th} and 26^{th}.

$$\therefore \quad T_{25} = 20 + (25-1)(-4) = 20 - 96 = -76$$

$$T_{26} = 20 + (26-1)(-4) = 20 - 100 = -80$$

S78. Since a, b, c are in A.P.

$$\therefore \quad 2b = a + c \qquad \text{... (i)}$$

Now, $\dfrac{1}{(\sqrt{b}+\sqrt{c})}, \dfrac{1}{(\sqrt{c}+\sqrt{a})}, \dfrac{1}{\sqrt{a}+\sqrt{b}}$ will be in A.P. if

$$\frac{1}{(\sqrt{c}+\sqrt{a})} - \frac{1}{(\sqrt{b}+\sqrt{c})} = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{(\sqrt{c}+\sqrt{a})}$$

$$\Rightarrow \quad \frac{(\sqrt{b}-\sqrt{a})}{(\sqrt{c}+\sqrt{a})(\sqrt{b}+\sqrt{c})} = \frac{(\sqrt{c}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{a})}$$

$$\Rightarrow \quad \frac{(\sqrt{b}-\sqrt{a})}{(\sqrt{b}+\sqrt{c})} = \frac{(\sqrt{c}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})} \quad \Rightarrow \quad 2b = a + c \qquad \text{... (ii)}$$

Which is true by (i)

Hence, $\dfrac{1}{(\sqrt{b}+\sqrt{c})}, \dfrac{1}{(\sqrt{c}+\sqrt{a})}, \dfrac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

S79. Let a be the first term and d be the common difference of given A.P.

Let $S_n = pn + qn^2$

Then, $\quad S_{n-1} = p(n-1) + q(n-1)^2$

$$\therefore \quad a_n = S_n - S_{n-1}$$

$$= (p_n + q_n)^2 - \{p(n-1) + q(n-1)^2\}$$

Thus, $\quad a_n = 2q_n + (p-q)$

$$a_{n-1} = 2q(n-1) + (p-q)$$

$$\therefore \quad d = a_n - a_{n-1} = \{2q_n + (p-q)\} - 2q(n-1) - (p-q)$$

$$= 2q \text{ which is a constant.}$$

Hence the common difference of the given A.P. is $2q$.

S80. Let $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

$\Rightarrow \quad b^2(c + a) - a^2(b + c) = c^2(a + b) - b^2(c + a)$

$\Rightarrow \quad c(b^2 - a^2) + ab(b - a) = a(c^2 - b^2) - b(c - b)$

$\Rightarrow \quad (b - a)(ab + bc + ca) = (c - b)(ab + bc + ca)$

$\Rightarrow \quad b - a = c - b$

$\Rightarrow \quad 2b = a + c$

$\Rightarrow$ a, b, c are in A.P.

$\Rightarrow$ $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are also in A.P.

S81. Given, $S_1 : 2, 7, 12, 17, ...$

$$T_{500} = 2 + (500 - 1)5 = 2497$$

$$S_2 = 1, 8, 15, 22, ...$$

$\Rightarrow \quad T_{300} = 1 + (300 - 1)7 = 2094$

Common difference of S_1 and S_2 are 5, and 7 respectively. Hence common difference of common term series is 35.

$\therefore$ A.P. of common terms is 22, 57, 92, ...

Let last term is 2094.

$\therefore \quad 2094 = 22 + (n - 1)35$

$\Rightarrow \quad n = 60.2$

$\because$ n is natural number

$\therefore \quad n = 60$

Then actual last common term

$$= 22 + (60 - 1)35 = 2062.$$

S82. Given, 2, 5, 8, 11, ... T_{60}

$$T_{60} = 2 + (60 - 1)3 = 179$$

Similarly,

$$T_{50} = 3 + (50 - 1)2 = 101$$

Hence last common term ≤ 101.

Now common difference of first A.P. is 3 and common difference of second A.P. is 2.

Hence, common difference of A.P. formed by common terms is L.C.M. of 3 and 2 which is 6. Also common terms are 5, 11, 17, 23, ...

for last term let $101 = 5 + (n - 1)6$

$\Rightarrow \quad n = 17$

$\therefore$ Total no. of identical terms = 17.

S83. Let the number of sides of the polygon be n.

Then,

Sum of interior angles = $(n-2)\pi$

Since, the difference between any two consecutive interior angles of the polygon is constant, its angle are in A.P.

$\because$ $a = 120°, \quad d = 5°$

Now $S_n = (n-2)\cdot 180°$

$$\Rightarrow \quad \frac{n}{2}\{2\cdot 120° + (n-1)5°\} = (n-2)\cdot 180°$$

$$\Rightarrow \quad \frac{235n}{2} + \frac{5n^2}{2} = 180°n - 360°$$

$$\Rightarrow \quad 5n^2 - 125n + 720 = 0$$

$$\Rightarrow \quad n^2 - 25n + 144 = 0$$

$$\Rightarrow \quad (n-9)(n-16) = 0$$

$$\Rightarrow \quad n = 9 \quad \text{or} \quad n = 16$$

But when n = 16 we have last angle = $\{120 + (16-1) \times 5\} = 195°$ which is not possible.

$\therefore$ $n = 9$

Hence the number of sides of the given polygon is equal to 9.

S84. Given, $a_1 = 3, \quad a_n = 3a_{n-1} + 2, \quad$ for all $n > 1$

For $n = 2$, $a_2 = 3a_{2-1} + 2$

$= 3a_1 + 2 = 3 \times 3 + 2 = 11$

For $n = 3$, $a_3 = 3a_2 + 2 = 3 \times 11 + 2 = 35$

For $n = 4$, $a_4 = 3a_3 + 2 = 3 \times 35 + 2 = 107$

For $n = 5$, $a_5 = 3a_4 + 2 = 3 \times 107 + 2 = 323$

Hence, the required sequence is

3, 11, 35, 107, 323, ...

Corresponding series

= 3 + 11 + 35 + 107 + 323 + ...

S85. Let a be the first term and d be the common difference of given A.P.

$$S_1 = \frac{n}{2}[2\cdot 1 + (n-1)\cdot 1] = \frac{n}{2}[n+1] \qquad \left[\because \; S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$S_2 = \frac{n}{2}[2 \cdot 1 + (n-1) \cdot 2] = \frac{n}{2}[2n] = n^2$$

$$S_3 = \frac{n}{2}[2 \cdot 1 + (n-1) \cdot 3] = \frac{n}{2}[3n - 1]$$

Now, $$S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}[3n-1]$$

$$= \frac{n}{2}(n + 1 + 3n - 1) = 2n^2 = 2S_2$$

S86. Let the angles be $a - 3d$, $a - d$, $a + d$, $a + 3d$.

Here, the common difference is $2d$.

i.e., $2d = 10°$

$\Rightarrow$ $d = 5°$

As, sum of all the angles in quadrilateral is 360°. We get

$a - 3d + a - d + a + d + a + 3d = 360° \quad \Rightarrow \quad a = 90°$

$\Rightarrow$ $a - 3d = 90° - 3 \times 5° = 75°$

$a - d = 90° - 5° = 85°$

$a + d = 90° + 5° = 95°$

$a + 3d = 90° + 3 \times 5° = 105°$

Hence, the angles are 75°, 85°, 95°, 105°.

S87. Let a be the first term and d be the common difference of given A.P.

$T_{p+1} = 2T_{q+1}$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow$ $a + pd = 2[a + qd]$

$\Rightarrow$ $a + (2q - p)d = 0$

$\Rightarrow$ $a = (p - 2q)d$

Now, $T_{3p+1} = a + 3pd$

$= (p - 2q)d + 3pd$

$= 2(2p - q)d$ ... (i)

$2T_{p+q+1} = 2[a + (p + q)d]$

$= 2[(p - 2q)d + (p + q)d]$

$= 2(2p - q)d$ ... (ii)

From Eq. (i) and (ii), we get

$$T_{3p+1} = 2T_{p+q+1}.$$

S88. We have,

$$(a_1 + a_2 + a_3 + a_4 + a_5) = \frac{1}{4}(a_6 + a_7 + a_8 + a_9 + a_{10})$$

$$\Rightarrow \quad (a_1 + a_2 + a_3 + a_4 + a_5) = \frac{1}{4}\{(a_1 + a_2 + ... + a_{10}) - (a_1 + a_2 + a_3 + a_4 + a_5)\}$$

$$\Rightarrow \quad S_5 = \frac{1}{4}(S_{10} - S_5)$$

$$\Rightarrow \quad 4S_5 = S_{10} - S_5$$

$$\Rightarrow \quad 5S_5 = S_{10}$$

$$\Rightarrow \quad 5 \times \frac{5}{2} \times [2 \times 2 + (5-1)d] = \frac{10}{2} \times [2 \times 2 + (10-1)d]$$

$$\Rightarrow \quad 50(1 + d) = (20 + 45d)$$

$$\Rightarrow \quad 5d = -30 \quad \Rightarrow \quad d = -6$$

$$\therefore \quad a_{20} = \{2 + (20-1) \times (-6)\} \quad \Rightarrow \quad a_{20} = -112$$

and $$S_{20} = \frac{20}{2} \times [2 \times 2 + (20-1) \times (-6)] = -1100$$

S89. Let a be the first term and d be the common difference of given A.P.

$$\because \quad a_3 + a_5 + a_8 = 11$$

$$\Rightarrow \quad a + 2d + a + 4d + a + 7d = 11$$

$$\Rightarrow \quad 3a + 13d = 11 \qquad \text{... (i)}$$

$$a_4 + a_2 = -2$$

$$\Rightarrow \quad a + 3d + a + d = -2$$

$$\Rightarrow \quad a = -1 - 2d \qquad \text{... (ii)}$$

Putting the value of a from Eq. (ii) in Eq. (i), we get

$$3(-1 - 2d) + 13d = 11, \quad \Rightarrow \quad 7d = 14$$

$$\Rightarrow \quad d = 2, \quad a = -5$$

$$a_1 + a_6 + a_7 = 7.$$

S90. Let a be the first term and d be the common difference of given A.P.

$$\because \quad \frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{5n+3}{3n+4}$$

Replace n by $(2n - 1)$

$$= \frac{2a + (2n-2)d}{2a' + (2n-2)d'} = \frac{5(2n-1)+3}{3(2n-1)+4}$$

$$= \frac{a+(n-1)d}{a'+(n-1)d'} = \frac{10n-2}{6n+1} \qquad \text{...... (i)}$$

putting n = 17 in equation (i), We get

$$= \frac{(a+(17-1)d)}{(a'+(17-1)d')} = \frac{168}{103}.$$

S91. The given numbers are in A.P.

Therefore,

$$2\log_4(2^{1-x}+1) = \log_2(5\times 2^x+1)+1$$

$$\Rightarrow \quad 2\log_2 2\left(\frac{2}{2^x}+1\right) = \log_2(5\times 2^x+1)+\log_2 2$$

$$\Rightarrow \quad \frac{2}{2}\log_2\left(\frac{2}{2^x}+1\right) = \log_2(5\times 2^x+1)2$$

$$\Rightarrow \quad \log_2\left(\frac{2}{2^x}+1\right) = \log_2(10\times 2^x+2)$$

$$\Rightarrow \quad \frac{2}{2^x}+1 = 10\times 2^x+2$$

Let $2^x = y$

$$\Rightarrow \quad \frac{2}{y}+1 = 10y+2$$

$$\Rightarrow \quad 10y^2+y-2=0 \quad \Rightarrow \quad (5y-2)(2y+1)=0$$

$$\Rightarrow \quad y = \frac{2}{5}, -\frac{1}{2}$$

$\because$ 2^x is always positive.

$$\therefore \quad 2^x = \frac{2}{5} \quad \Rightarrow \quad x = \log_2(2/5)$$

$$\therefore \quad x = \log_2 2 - \log_2 5$$

$$\Rightarrow \quad x = 1-\log_2 5.$$

S92. The smallest and largest number of three digits which are divisible by 7 is 105 and 994 respectively. So the sequence of three digit numbers which are divisible by 7 is 105, 112, 119, ... 994.

$$\because \quad a = 105,\ d = 7,$$

$$\therefore \quad a_n = 994$$

$$994 = a+(n-1)d$$

$$\Rightarrow \quad 994 = 105+(n-1)7$$

$\Rightarrow \quad n = 128$

Now, required sum is

$$= \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{128}{2}[2 \times 105 + (128-1) \times 7]$$

$$= 70336.$$

S93. Let a be the first term and d be the common difference of given A.P.

Let the digits at ones, tens and hundreds place be $(a-d)$, a and $(a+d)$ respectively.

Then the number is

$(a+d) \times 100 + a \times 10 + (a-d) = 111a + 99d$

The number obtained by reversing the digit is

$(a-d) \times 100 + a \times 10 + (a+d) = 111a - 99d$

$$(a-d) + a + (a+d) = 15 \quad \text{... (i)}$$

$$3a = 15 \Rightarrow a = 5$$

and $111a - 99d = 111a + 99d - 594$

$\Rightarrow \quad 198d = 594 \quad \text{... (ii)}$

$\Rightarrow \quad a = 5, \quad d = 3$

So, the number is $111 \times 5 + 99 \times 3 = 852$

S94. **For First A.P. :**

Let First term $= a_1$

Common difference $= d_1$

and S_{n_1} = Sum of n terms

For Second A.P. :

Let First term $= a_2$

Common difference $= d_2$

and S_{n_2} = Sum of n terms

Given, $\dfrac{S_{n_1}}{S_{n_2}} = \dfrac{5n+4}{9n+6}$

Required to find $\dfrac{T_{1(18)}}{T_{2(18)}} = \dfrac{a_1 + 17d_1}{a_2 + 17d_2}$

We have, $$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{5n+4}{9n+6}$$

or $$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{5n+4}{9n+6}$$

or $$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{5n+4}{9n+6}$$

Taking $$\frac{n-1}{2} = 17$$

or $$n = 34 + 1 = 35$$

We have, $$\frac{a_1 + 17d_1}{a_2 + 17d_2} = \frac{5 \times 35 + 4}{9 \times 35 + 6} = \frac{179}{321}.$$

S95. Let a be the first term and d be the common difference of given A.P.

Given, $$T_k = 5k + 1$$

$$T_1 = 5 \times 1 + 1 = 6$$

$$T_2 = 5 \times 2 + 1 = 11$$

$$T_3 = 5 \times 3 + 1 = 16$$

$$T_4 = 5 \times 4 + 1 = 21$$

Now, $$a = 6$$

$$d = 11 - 6 = 5$$

Thus, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 6 + (n-1)(5)]$$

$$= \frac{n}{2}[12 + 5n - 5]$$

$$= \frac{n}{2}[5n + 7] = \frac{5n^2}{2} + \frac{7n}{2}.$$

S96. Let a be the first term and d be the common difference of given A.P.

Given: A.P. : 25, 22, 19, ...

Here, $$S_n = 116$$

$$a = 25$$

$$d = 22 - 25 = -3$$

i.e., Last term = T_n

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

or $116 = \frac{n}{2}[2 \times 25 + (n-1)(-3)]$

or $232 = n[50 - 3n + 3]$

or $n[53 - 3n] = 232$

or $3n^2 - 53n + 232 = 0$

or $3n^2 - 29n - 24n + 232 = 0$

or $n(3n - 29) - 8(3n - 29) = 0$

or $(3n - 29)(n - 8) = 0$

or $n = 8 \quad \text{as} \quad n \in N$

Now Last term $= T_n = a + (n-1)d$

or $T_8 = 25 + (8-1)(-3)$

$= 25 - 21 = 4.$

S97. Let a be the first term and d be the common difference of given A.P.

Here, $a = -6$

$$d = -\frac{11}{2} - (-6) = \frac{-11}{2} + 6$$

$$= \frac{-11+12}{2} = \frac{1}{2}$$

Let n be the number of terms so that

$$S_n = -25$$

Since $S_n = \frac{n}{2}[2a + (n-1)d]$

or $-25 = \frac{n}{2}\left[2 \times (-6) + (n-1) \times \frac{1}{2}\right]$

or $-25 = \frac{n}{2}\left[-12 + \frac{n-1}{2}\right]$

or $-25 = \frac{n}{2}\left[\frac{-24 + n - 1}{2}\right]$

or $-25 = \frac{n}{2}\left(\frac{n-25}{2}\right)$

or $-100 = n(n - 25)$

or $n^2 - 25n + 100 = 0$

or $(n-5)(n-20) = 0$

or $n = 5$ or $n = 20$

Both values of n give the required sum.

S98. Let a be the first term and d be the common difference of given A.P.

The numbers between 100 and 1000 which are multiple of 5 are 105, 110, 115, ..., 995.

Now, $a = 105$

$d = 110 - 105 = 5$

and $l = T_n = 995$

Since, $T_n = a + (n-1)d$

$995 = 105 + (n-1)5$

or $995 - 105 = (n-1)5$

or $(n-1) = \dfrac{890}{5} = 178$

or $n = 178 + 1 = 179$

Now, $S_n = \dfrac{n}{2}[a + l] = \dfrac{179}{2}[105 + 995]$

$= \dfrac{179}{2} \times 1100 = 179 \times 550 = 98450.$

S99. Let a be the first term and d be the common difference of given A.P.

Here, $S_n = 1 + 3 + 5 + 7 + ... + 2001$

Now, $a = 1$

$d = 3 - 1 = 2, \quad l = 2001$

Let $t_n = 2001$

Therefore, $a + (n-1)d = 2001$

or $1 + (n-1)(2) = 2001$

or $2(n-1) = 2000$

or $n - 1 = 1000$

or $n = 1001$

We have, $S_n = \dfrac{n}{2}[a + l]$

or $S_{1001} = \dfrac{1001}{2}[1 + 2001]$

$= \dfrac{1001}{2}[2002]$

$= 1001 \times 1001 = 1002001.$

S100. Given, $a_1 = -1, \quad a_n = \frac{a_{n-1}}{n}, \quad n \geq 2$

For $n = 2$, $a_2 = \frac{a_1}{2} = \frac{-1}{2}$

For $n = 3$, $a_3 = \frac{a_2}{3} = \frac{-\frac{1}{2}}{3} = -\frac{1}{6}$

For $n = 4$, $a_4 = \frac{a_3}{4} = \frac{-\frac{1}{6}}{4} = -\frac{1}{24}$

For $n = 5$, $a_5 = \frac{a_4}{5} = \frac{-\frac{1}{24}}{5} = -\frac{1}{120}$

Hence, first five terms are

$$-1, \frac{-1}{2}, -\frac{1}{6}, -\frac{1}{24}, -\frac{1}{120}$$

and the series is

$$(-1) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots = -1 - \frac{1}{2} - \frac{1}{6} - \frac{1}{24} - \frac{1}{120} - \ldots$$

S101. 2 digit numbers when divided by 4, yielding 1 as remainder are 13, 17, 21, ..., 97

Now, $a = 13$

$d = 17 - 13 = 4$

Let $T_n = 97$

$\Rightarrow$ $a + (n - 1)d = 97$

or $13 + (n - 1)(4) = 97$

or $(n - 1) \times 4 = 84$

or $n - 1 = 21$

or $n = 22$

Now, $S_n = \frac{n}{2}[a + l]$

$S_{22} = \frac{22}{2}(13 + 97)$

$= 11 \times 110 = 1210.$

S102. The numbers between 200 and 400 which are divisible by 7 are 203, 210, 217, 224, ..., 399

Here $a = 203$

$d = 7$

$l = T_n = 399$

$$T_n = a + (n-1)\,d$$

or $$399 = 203 + (n-1)\,7$$

or $$399 - 203 = (n-1)\,7$$

or $$196 = (n-1)\,7$$

or $$(n-1) = \frac{196}{7} = 28$$

or $$n = 28 + 1 = 29$$

i.e., there are 29 numbers between 200 and 400 which are divisible by 7.

Now, $$S_n = \frac{n}{2}(a + l)$$

or $$S_n = \frac{29}{2}(203 + 399)$$

$$= \frac{29}{2} \times 602$$

$$= 29 \times 301 = 8729.$$

S103. Let three numbers in A.P. be

$$a - d,\quad a,\quad a + d.$$

Their sum = $(a - d) + a + (a + d) = 24$

or $$3a = 24$$

or $$a = 8$$

Their product = $(a - d)\,(a)\,(a + d) = 440$

or $$a\,(a^2 - d^2) = 440$$

or $$8\,(64 - d)^2 = 440$$

or $$64 - d^2 = \frac{440}{8} = 55$$

or $$d^2 = 64 - 55 = 9$$

or $$d = \pm 3$$

Hence, the numbers are 8 – 3, 8, 8 + 3 or 8 + 3, 8, 8 – 3, *i.e.*, 5, 8, 11 or 11, 8, 5.

S104. Let a be the first term and d the common difference of an A.P.

Now, we want to prove that

$$T_{m+n} + T_{m-n} = 2T_m$$

$$\text{L.H.S.} = T_{m+n} + T_{m-n}$$

$$= [a + (m + n - 1)\,d] + [a + (m - n - 1)\,d]$$

$$= 2a + (m + n - 1 + m - n - 1)\,d$$

$= 2a + (2m - 2)\,d$

$= 2\,[a + (m - 1)\,d]$

$= 2\,T_m$ = R.H.S.

S105. (i) Here, $a_n = 2n + 5$

Substituting $n = 1, 2, 3$, we get

$a_1 = 2\,(1) + 5 = 7, \quad a_2 = 9, \quad a_3 = 11$

Therefore, the required terms are 7, 9 and 11.

(ii) Here, $a_n = \dfrac{n-3}{4}$

Thus, $a_1 = \dfrac{1-3}{4} = -\dfrac{1}{2}, \quad a_2 = -\dfrac{1}{4}, \quad a_3 = 0$

Hence, the first three terms are $-\dfrac{1}{2}, -\dfrac{1}{4}$ and 0.

S106. Given, $S_n = 3n^2 + 5n$

$S_1 = 3\,(1)^2 + 5 \times 1 = 8$

$\Rightarrow \quad T_1 = 8$

$S_2 = 3\,(2)^2 + 5 \times 2 = 22$

Thus, $T_1 + T_2 = 22$

or $8 + T_2 = 22$

or $T_2 = 14$

Now, $a = 8$,

$d = 14 - 8 = 6$

As $T_m = 164$

or $a + (m - 1)\,d = 164$

or $8 + (m - 1)\,(6) = 164$

or $8 + 6m - 6 = 164$

or $6m = 162$

or $m = \dfrac{162}{6} = 27$

Hence, $m = 27$.

S107. Since, $a\left(\dfrac{1}{b}+\dfrac{1}{c}\right), b\left(\dfrac{1}{c}+\dfrac{1}{a}\right), c\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ are in A.P.

$\Rightarrow \quad \dfrac{ac+ab}{bc}, \dfrac{ab+bc}{ca}, \dfrac{cb+ac}{ab}$ are in A.P.

$\Rightarrow \quad \frac{ac+ab}{bc}+1, \frac{ab+bc}{ca}+1, \frac{bc+ac}{ab}+1$ are also in A.P.

(adding 1 to each term)

$\Rightarrow \quad \frac{ac+ab+bc}{bc}, \frac{ab+bc+ca}{ca}, \frac{bc+ac+ab}{ab}$ are also in A.P.

$\Rightarrow \quad \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

(Dividing each term by $ab + bc + ca$)

$\Rightarrow \quad \frac{abc}{bc}, \frac{abc}{ca}, \frac{abc}{ab}$ are also in A.P.

(Multiplying each term by abc)

$\Rightarrow \quad a, b, c$ are in A.P. **Proved.**

S108. We have, $a_1 = 1$

$a_2 = a_1 + 2 = 1 + 2 = 3$

$a_3 = a_2 + 2 = 3 + 2 = 5$

$a_4 = a_3 + 2 = 5 + 2 = 7$

$a_5 = a_4 + 2 = 7 + 2 = 9$

Hence, the first five terms of the sequence are 1, 3, 5, 7 and 9. Hence, the corresponding series is 1 + 3 + 5 + 7 + 9 + ...

S109. Given, $a = 11$

Let the first four terms of the A.P. be

$$a, a + d, a + 2d, a + 3d, ..., T_n$$

Now, $a + (a + d) + (a + 2d) + (a + 3d) = 56$

or $4a + 6d = 56$

or $2a + 3d = 28$

and $T_n + T_{n-1} + T_{n-2} + T_{n-3} = 112$

$$[a + (n - 1) d] + [a + (n - 2) d] + [a + (n - 3) d] + [a + (n - 4) d] = 112$$

$$4a + 4nd - 10d = 112$$

$$2a + 2nd - 5d = 56$$

$\because \quad 2a + 3d = 28$

$$2 \times 11 + 2nd - 5d = 56$$

$$2nd - 5d = 34$$

$$2a + 3d = 28$$

$$2 \times 11 + 3d = 28$$
$$3d = 6$$
$$d = 2$$

$\Rightarrow$ $$2n \times 2 - 5 \times 2 = 34$$

or $$4n = 44$$

$\Rightarrow$ $$n = 11$$

Hence, the number of terms are 11.

S110. Given that

$$(a^2 + b^2 + c^2)\,p^2 - 2\,(ab + bc + cd)\,p + (b^2 + c^2 + d^2) \le 0 \qquad \text{... (i)}$$

But L.H.S.. $= (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2)$,

which gives $$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0 \qquad \text{... (ii)}$$

Since, the sum of squares of real numbers is not negative, therefore, from Eq. (i) and (ii)

we have, $$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$$

or $$ap - b = 0, \quad bp - c = 0, \quad cp - d = 0$$

This implies that $$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$$

Hence, a, b, c and d are in G.P.

S111. Let $\frac{ab + ac}{bc}, \frac{bc + ba}{ca}, \frac{ca + bc}{ab}$ are in A.P.

$\Rightarrow$ $\frac{a(b + c)}{bc}, \frac{b(c + a)}{ca}, \frac{c(a + b)}{ab}$ are in A.P.

$\Rightarrow$ $$\frac{b(c + a)}{ca} - \frac{a(b + c)}{bc} = \frac{c(a + b)}{ab} - \frac{b(c + a)}{ca}$$

$\Rightarrow$ $$\frac{b^2(c + a) - a^2(b + c)}{abc} = \frac{c^2(a + b) - b^2(c + a)}{abc}$$

$\Rightarrow$ $$b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2c - b^2a$$

$\Rightarrow$ $$(b^2c - a^2c) + (b^2a - a^2b) = (c^2a - b^2a) + (c^2b - b^2c)$$

$\Rightarrow$ $$c\,(b^2 - a^2) + ab\,(b - a) = a\,(c^2 - b^2) + bc\,(c - b)$$

$\Rightarrow$ $$(b - a)\,(cb + ca + ab) = (c - b)\,(cb + ca + ab)$$

$\Rightarrow$ $$b - a = c - b$$

$\Rightarrow$ $$2b = a + c$$

a, b, c are in A.P.

Hence, the problem.

S112. Let a be the first term and d be the common difference of given A.P.

Here, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$$\Rightarrow \quad \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\Rightarrow \quad m[2a + (n-1)d] = n[2a + (m-1)d]$$

$$\Rightarrow \quad 2(m-n)a = (m-n)d \quad \Rightarrow \quad d = 2a$$

Now, $\frac{t_m}{t_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$$\frac{a+(m-1)(2a)}{a+(n-1)(2a)} = \frac{2m-1}{2n-1}.$$

S113. Let $S_n = \frac{n}{2}[2a + (n-1)d]$; $S'_n = \frac{n}{2}[2A + (n-1)d]$

Now, $\frac{S_n}{S'_n} = \frac{7n+1}{4n+27}$

$$\Rightarrow \quad \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{a+\frac{n-1}{2}}{A+\frac{n-1}{2}D} = \frac{7n+1}{4n+27} \quad \text{... (i)}$$

We have to find the ratio of the 11^{th} term, *i.e.*, $\frac{a+10d}{A+10D}$

From (i) $\frac{n-1}{2} = 10 \quad \Rightarrow \quad n = 21$

$$\Rightarrow \quad \frac{a+10d}{A+10D} = \frac{7(21)+1}{4(21)+27} = \frac{148}{111}$$

Hence, the ratio of the 11th terms is 148 : 111

S114. Here, $t_p = a$, $t_q = b$, $t_r = c$

$\Rightarrow$ $\frac{p}{2}[2A + (p-1)D] = a$

$\Rightarrow$ $\frac{q}{2}[2A + (q-1)D] = b$

$\Rightarrow$ $\frac{r}{2}[2A + (r-1)D] = c$

Now, $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$

$$= \frac{(q-r)}{p}\cdot\frac{p}{2}[2A+(p-1)D] + \frac{r-p}{q}\cdot\frac{q}{2}[2A+(q-1)D] + \frac{p-q}{r}\cdot\frac{r}{2}[2A+(r-1)D]$$

$$= A[q-r+r-p+p-q] + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= A(0) + \frac{D}{2}(0) = 0.$$

S115. Let A be the first term and D be the common difference of given A.P.

Here, $t_p = a$, $t_q = b$

$\Rightarrow$ $A + (p-1)D = a$ ... (i)

$A + (q-1)D = b$ ... (ii)

Subtracting Eq. (ii) from Eq. (i), we get

$(p-q)D = a - b$

$\Rightarrow$ $D = \frac{a-b}{p-q}$

Adding Eq. (i) and Eq. (ii), we get

$2A + (p+q-2)D = a + b$

$\Rightarrow$ $2A + (p+q-1)D = a + b + D$

$2A + (p+q-1)D = a + b + \frac{a-b}{p-q}$ ... (iii)

Now, $S_{p+q} = \frac{p+q}{2}[2A + (p+q-1)D]$

$= \frac{p+q}{2}\left[a + b + \frac{a-b}{p-q}\right]$. [from (iii)]

S116. Here, $t_1 = a$, $t_2 = b$, $t_n = c$, $d = t_2 - t_1 = b - a$, $t_n = a + (n-1)d$

$\Rightarrow$ $c = a + (n-1)(b-a)$

$\Rightarrow$ $c - a = (n-1)(b-a)$

$\Rightarrow \quad n - 1 = \dfrac{c-a}{b-a}$

$\Rightarrow \quad n = \dfrac{c-a}{b-a} + 1 = \dfrac{c-a+b-a}{b-a}$

$= \dfrac{b+c-2a}{b-a}$

Now, $S_n = \dfrac{n}{2}[a + t_n]$ $\left[\because \ S_n = \dfrac{n}{2}(a_1 + a_n) = \dfrac{n}{2}(a_2 + a_{n-1}) = \cdots = \dfrac{n}{2}(a_k + a_{n-k+1})\right]$

$= \dfrac{(b+c-2a)(a+c)}{2(b-a)}.$ $[k \in N]$

S117. The given A.P.'s are (1, 2, 3, ...), (2, 5, 8, ...), (3, 8, 13, ...) ... (m, $3m - 1$, $5m - 2$, ...)

$\Rightarrow \quad S_1 = \dfrac{n}{2}[2 \cdot 1 + (n-1) \cdot 1]$

$S_2 = \dfrac{n}{2}[2 \cdot 2 + (n-1) \cdot 3]$

$S_3 = \dfrac{n}{2}[2 \cdot 3 + (n-1) \cdot 5]$

$\vdots$

$S_m = \dfrac{n}{2}[2 \cdot m + (n-1)(2m-1)]$

$\Rightarrow \quad S_1 + S_2 + S_3 + \ldots + S_m$

$= \dfrac{n}{2}[2(1 + 2 + 3 + \ldots + m) + (n-1)\{1 + 3 + 5 + \ldots + (2m-1)\}]$

$= \dfrac{n}{2}\left[2 \cdot \dfrac{m(m+1)}{2} + (n-1) \cdot \dfrac{m}{2}(1 + 2m - 1)\right]$

$= \dfrac{n}{2}[m(m+1) + (n-1)m^2]$

$= \dfrac{mn}{2}(mn+1).$

S118. $\dfrac{1}{\sqrt{a_1} + \sqrt{a_2}} + \dfrac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots + \dfrac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$

$= \dfrac{\sqrt{a_2} - \sqrt{a_1}}{(\sqrt{a_2} + \sqrt{a_1})(\sqrt{a_2} - \sqrt{a_1})} + \dfrac{\sqrt{a_3} - \sqrt{a_2}}{(\sqrt{a_3} + \sqrt{a_2})(\sqrt{a_3} - \sqrt{a_2})} + \ldots + \dfrac{\sqrt{a_n} - \sqrt{a_{n-1}}}{(\sqrt{a_n} + \sqrt{a_{n-1}})(\sqrt{a_n} - \sqrt{a_{n-1}})}$

$= \dfrac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \dfrac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \ldots + \dfrac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}}$ $[\because \ d = a_2 - a_1 = a_3 - a_2 = \ldots]$

$$= \frac{1}{d}[(\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2}) + \ldots + (\sqrt{a_n} - \sqrt{a_{n-1}})]$$

$$= \frac{1}{d}[\sqrt{a_n} - \sqrt{a_1}] \cdot \frac{\sqrt{a_n} + \sqrt{a_1}}{\sqrt{a_n} + \sqrt{a_1}} = \frac{a_n - a_1}{d[\sqrt{a_n} + \sqrt{a_1}]}$$

$$= \frac{a_1 + (n-1)d - a_1}{d[\sqrt{a_n} + \sqrt{a_1}]} \qquad [\because \quad t_n = a + (n-1)d]$$

$$= \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}.$$

S119. Let a be the first term and d be the common difference of given A.P.

Here, $t_m = \frac{1}{n}$, $t_n = \frac{1}{m}$

$$\Rightarrow \qquad a + (m-1)d = \frac{1}{n}$$

$$a + (n-1)d = \frac{1}{m}$$

Solving the equations, we get

$$a = \frac{1}{mn}, \quad d = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}[2a + (mn-1)d]$$

$$= \frac{mn}{2}\left[\frac{2}{mn} + \frac{mn-1}{mn}\right] = \frac{mn+1}{2}$$

S120. We have, $d = a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1}$.

From L.H.S.

$$\sin d\,[\text{cosec } a_1 \text{ cosec } a_2 + \text{cosec } a_2 \text{ cosec } a_3 + \ldots + \text{cosec } a_{n-1} \text{ cosec } a_n]$$

$$= \frac{\sin d}{\sin a_1 \sin a_2} + \frac{\sin d}{\sin a_2 \sin a_3} + \ldots + \frac{\sin d}{\sin a_{n-1} \sin a_n}$$

$$= \frac{\sin(a_2 - a_1)}{\sin a_1 \sin a_2} + \frac{\sin(a_3 - a_2)}{\sin a_2 \sin a_3} + \ldots + \frac{\sin(a_n - a_{n-1})}{\sin a_{n-1} \sin a_n}$$

$$= \frac{\sin a_2 \cos a_1 - \cos a_2 \sin a_1}{\sin a_1 \sin a_2} + \frac{\sin a_3 \cos a_2 - \cos a_3 \sin a_2}{\sin a_2 \sin a_3} + \ldots$$

$$+ \frac{\sin a_n \cos a_{n-1} - \cos a_n \sin a_{n-1}}{\sin a_{n-1} \cdot \sin a_n}$$

$= (\cot a_1 - \cot a_2) + (\cot a_2 - \cot a_3) + ... + (\cot a_{n-1} - \cot a_n)$

$= \cot a_1 - \cot a_n$. **R.H.S.**

S121. Let $G_1 = (1)$, $G_2 = (2, 3, 4)$, $G_3 = (5, 6, 7, 8, 9)$

Number of terms in G_1, G_2, G_3, ... are 1, 3, 5, ... respectively.

$\Rightarrow$ The number of terms in G_n will be $2n - 1 = N$.

Now, t_1 of $G_1 = 1 = 1 + (1 - 1)^2$

t_1 of $G_2 = 2 = 1 + (2 - 1)^2$

t_1 of $G_3 = 5 = 1 + (3 - 1)^2$

t_1 of $G_4 = 10 = 1 + (4 - 1)^2$

Hence, t_1 of $G_n = 1 + (n - 1)^2$,

$= n^2 - 2n + 2 = A$

Hence, the sum of the terms in the n^{th} group.

$$= \frac{N}{2}[2A + (N - 1)D]$$

$$= \frac{2n-1}{2}[2(n^2 - 2n + 2) + (2n - 1 - 1)\cdot 1] \qquad [\because \; D = 1]$$

$$= \frac{2n-1}{2}[2n^2 - 2n + 2]$$

$= (2n - 1)(n^2 - n + 1)$

$= 2n^3 - 3n^2 + 3n - 1$

$= n^3 + (n^3 - 3n^2 + 3n - 1)$

$= n^3 + (n - 1)^3$

$= (n - 1)^3 + n^3$.

S122. a^2, b^2, c^2 are in A.P.

$$2b^2 = a^2 + c^2 \qquad ... (i)$$

(a) Now, $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ will in in A.P.

If $$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$\Rightarrow$ $$\frac{b-a}{(c+a)(b+c)} = \frac{c-b}{(a+b)(c+a)}$$

$$\Rightarrow \quad \frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$\Rightarrow \quad b^2 - a^2 = c^2 - b^2$$

$\Rightarrow \quad 2b^2 = a^2 + c^2$, which is true from (i)

$\Rightarrow \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

(b) Now, $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in A.P.

If $\quad \frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a}$

$$\Rightarrow \quad \frac{b(b+c)-a(c+a)}{(c+a)(b+c)} = \frac{c(c+a)-b(a+b)}{(a+b)(c+a)}$$

$$\Rightarrow \quad \frac{(b^2-a^2)+c(b-a)}{b+c} = \frac{(c^2-b^2)+a(c-b)}{a+b}$$

$$\Rightarrow \quad \frac{(b-a)(b+a+c)}{c+b} = \frac{(c-b)(c+b+a)}{b+a}$$

$$\Rightarrow \quad \frac{b-a}{c+b} = \frac{c-b}{b+a}$$

$$\Rightarrow \quad b^2 - a^2 = c^2 - b^2$$

$\Rightarrow \quad 2b^2 = a^2 + c^2$ which is true from (i)

Hence, $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

S123.(a) Here, $a = 1$, $d = 3$, $S_n = 590$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 590 = \frac{n}{2}[2(1) + (n-1)3]$$

$\Rightarrow \quad n(3n-1) = 1180 \quad \Rightarrow \quad 3n^2 - n - 1180 = 0$

$$\Rightarrow \quad n = 20, -\frac{59}{3}$$

$\Rightarrow \quad n = 20 \qquad [\because \; n \in N]$

(b) Here, $a = 20$, $d = -2$, $S_n = 0$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad 0 = \frac{n}{2}[2(20) + (n-1)(-2)]$

$\Rightarrow \quad 0 = n[21 - n]$

$\Rightarrow \quad n = 0, 21$

$\Rightarrow \quad n = 21 \quad [\because \ n \in N]$

(c) Here, $a = 16$, $d = -2$, $S_n = 60$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad 60 = \frac{n}{2}[2(16) + (n-1)(-2)]$

$\Rightarrow \quad 60 = n[17 - n]$

$\Rightarrow \quad n^2 - 17n + 60 = 0 \quad n = 5, 12$

The sum of the first five terms is 60 and the sum of next seven terms is zero this is the reason to get two values of n.

(d) Here, $a = 54$, $d = -3$, $S_n = 513$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad 513 = \frac{n}{2}[2(54) + (n-1)(-3)]$

$\Rightarrow \quad 1026 = n[111 - 3n]$

$\Rightarrow \quad 3n^2 - 111n + 1026 = 0$

$\Rightarrow \quad n^2 - 37n + 342 = 0$

$\Rightarrow \quad n = 18, 19$

Here, $S_{18} = 513$, $t_{19} = 0$, $S_{19} = S_{18} + t_{19} = S_{18}$, which is the reason to get two values of n.

S124. (a) Here, $a = 5$, $d = 8$, $t_n = 189$

$t_n = a + (n-1)d \Rightarrow 189 = 5 + (n-1)(8) \Rightarrow n = 24$

Now, $S_n = \frac{n}{2}[a + t_n] = \frac{24}{2}[5 + 189] = 2328$

(b) Here, $a = 6$, $d = 4$, $t_n = 106$

$t_n = a + (n-1)d \Rightarrow 106 = 6 + (n-1)(4) \Rightarrow n = 26$

Now, $S_n = \frac{n}{2}[a + t_n] = \frac{26}{2}[6 + 106] = 1456$

(c) Here, $a = 1$, $d = 1$, $t_n = n$

$$S_n = \frac{n}{2}[a + t_n] = \frac{n}{2}[1 + n] = \frac{n(n+1)}{2}$$

(d) $\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots + \log \frac{a^n}{b^{n-1}}$

$= \log a + [2 \log a - \log b] + [3 \log a - 2 \log b] + \ldots + [n \log a - (n-1) \log b]$

$= [\log a + 2 \log a + 3 \log a + \ldots + n \log a] - [\log b + 2 \log b + \ldots + (n-1) \log b]$

$= \log a\, [1 + 2 + 3 + \ldots + n] - \log b\, [1 + 2 + 3 \ldots + (n-1)]$

$= \frac{n(n+1)}{2} \log a - \frac{n(n-1)}{2} \log b$

$= \frac{n}{2} [(n+1)\log a - (n-1) \log b]$

$= \frac{n}{2} [n(\log a - \log b) + \log a + \log b]$

$= \frac{n}{2}\left[n \log \frac{a}{b} + \log ab \right].$

S125. Sum = (2 + 4 + 6 + ... + 100) + (5 + 10 + 15 + ... + 100) – (10 + 20 + 30 + ... + 100)

Sum = $S_1 + S_2 - S_3$

For First Series: $a = 2,$

$d = 2,$

$T_n = 100$

$T_n = a + (n-1)d$

$100 = 2 + (n-1)2$

$98 = (n-1) \times 2$

$2n - 2 = 98$

$2n = 100$

$\Rightarrow$ $n = 50$

Thus, $S_{50} = \frac{50}{2}[2 + 100] = 50 \times 51 = 2550.$

For Second Series: $a = 5,$

$d = 5,$

$T_n = 100$

Therefore, $a + (n-1)d = 100$

$5 + (n-1) \times 5 = 100$

$5n = 100$

$\Rightarrow$ $n = 20$

Thus, $S_{20} = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050.$

For Third Series: $a = 10,$

$d = 10,$

$T_n = 100$

Therefore, $a + (n - 1)\,d = 100$

$10 + (n - 1) \times 10 = 100$

$10n = 100$

$\Rightarrow \quad n = 10$

Thus, $S_{10} = \frac{10}{2}[10 + 100] = 5 \times 110 = 550.$

Hence, Required sum = 2550 + 1050 – 550 = 3050.

Date: 12/10/2021

Q1. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

Q2. Insert 6 arithmetic means between 3 and 24.

Q3. a, b and c are three numbers in A.P. If x is the arithmetic mean of a and b and y is the arithmetic mean of b and c, then prove that the arithmetic mean of x and y is b.

Q4. After inserting x A.M.'s between 2 and 38, the sum of the resulting progression is 200. Then find the value of x.

Q5. If a_1, a_2, a_3, a_4, a_5 and a_6 are six arithmetic means between 3 and 31, then find the value of $a_6 - a_5$ and $a_1 + a_6$.

Q6. Insert: (i) 3 arithmetic mean between 2 and 10; (ii) 4 arithmetic mean between 1 and 16; and (iii) 5 arithmetic mean between 3 and 27.

Q7. If the A.M. between p^{th} and q^{th} terms of an A.P. be equal to the A.M. between the r^{th} and s^{th} terms of the A.P., show that $p + q = r + s$.

Q8. If 13 A. M.'s are inserted between 52 and 17, then find the number of A.M.'s. Which contains integral values?

Q9. If m A. M.'s. are inserted between 5 and 181 in such a way that 11^{th} A.M. is 93, then find the value of m.

Q10. n arithmetic means are inserted between 3 and 17. If the ratio of the last and the first arithmetic mean is 3 : 1, then find the value of n.

Q11. Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

Q12. Insert 5 arithmetic means between 4 and 22.

Q13. Between 1 and 31, m numbers have been inserted in such a way that the ratio of 7^{th} and $(m-1)^{th}$ numbers is 5 : 9. Find the value of m.

Q14. Show that the sum of n arithmetic means between two numbers is n times the single arithmetic mean between them.

Q15. If a_1, a_2, ..., a_n are arithmetic means between a and b, then find the value of $2\sum_{i=1}^{n} a_i$.

Q16. If n arithmetic means are inserted between 20 and 80 such that the ratio of first mean to the last mean is 1 : 3, then find the value of n.

Q17. Find the A.M. between

(i) 6 and 12 (ii) 5 and 22

(iii) $(\cos\theta + \sin\theta)^2$ and $(\cos\theta - \sin\theta)^2$ (iv) $(x + y)^2$ and $(x - y)^2$

Q18. Insert 8 A.Ms. between 2 and 29. Also, verify that the sum of these 8 A.Ms. is equal to 8 times the A.M. between 2 and 29.

Date: 12/10/2021

S1. We know that A.M. between a and $b = \dfrac{a+b}{2}$

$\Rightarrow \quad \dfrac{a^n + b^n}{a^{n-1} + b^{n-1}} = \dfrac{a+b}{2}$ [Given]

$\Rightarrow \quad 2a^n + 2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$

$\Rightarrow \quad a^n - a^{n-1}b = ab^{n-1} - b^n$

$\Rightarrow \quad a^{n-1}(a-b) = b^{n-1}(a-b)$

$\Rightarrow \quad a^{n-1} = b^{n-1}$ [$\because a \neq b$]

$\Rightarrow \quad \left(\dfrac{a}{b}\right)^{n-1} = 1 = \left(\dfrac{a}{b}\right)^0$ $\left(\because \left(\dfrac{a}{b}\right)^0 = 1\right)$

$\Rightarrow \quad n - 1 = 0 \quad \Rightarrow \quad n = 1$

S2. Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six A.Ms. between 3 and 24.

$\therefore \quad 3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ are in A.P

We have $T_8 = 24 \quad \Rightarrow \quad 3 + (8-1)d = 24$ [$\because T_n = a + (n-1)d$]

$\Rightarrow \quad 3 + 7d = 24 \quad \Rightarrow \quad d = 3$ ($a = 3$)

$\therefore \quad A_1 = a + d = 3 + 3 = 6$

$A_2 = A_1 + d = 6 + 3 = 9$

$A_3 = A_2 + d = 9 + 3 = 12$

$A_4 = A_3 + d = 12 + 3 = 15$

$A_5 = A_4 + d = 15 + 3 = 18$

$A_6 = A_5 + d = 18 + 3 = 21$

S3. x is the arithmetic mean of a and $b \Rightarrow x = \dfrac{a+b}{2}$ ... (i)

y is the arithmetic mean of b and $c \Rightarrow y = \dfrac{b+c}{2}$ ... (ii)

$$\therefore \quad \frac{x+y}{2} = \frac{\left(\frac{a+b}{2}\right) + \left(\frac{b+c}{2}\right)}{2} = \frac{a+2b+c}{4} = \frac{4b}{4} = b.$$

[$\because$ a, b, c are in A. P. $\Rightarrow a + c = 2b$]

Hence, the arithmetic mean of x and y is b.

S4. Let $a_1, a_2, \ldots, a_x$ be the x A.M.s. between 2 and 38.

The sum of these x A.M.s. = $x \times$ (Single A.M. between 2 and 38)

$$\Rightarrow \quad (a_1 + a_2 + \ldots + a_x) = x\left(\frac{2+38}{2}\right) = 20x.$$

Now, the resulting progression is 2, a_1, a_2, ... a_x, 38.

We have: $2 + a_1 + a_2 + \ldots. + a_x + 38 = 200$

$\Rightarrow \quad a_1 + a_2 + \ldots + a_x = 160$

$\Rightarrow \quad 20x = 160 \quad \Rightarrow \quad x = 8.$

S5. Let a be the first term and d be the common difference of given A.P.

$a_1, a_2, a_3, a_4, a_5, a_6$ are six arithmetic means between 3 and 31

$\Rightarrow \quad 3, a_1, a_2, a_3, a_4, a_5, a_6, 31$ are A.P.

Let d be the common difference of this A.P. 31 is the 8^{th} term of the A.P.

i.e., $t_8 = 31 \quad \Rightarrow \quad 3 + 7d = 31 \quad \Rightarrow \quad d = 4.$ [***Note:*** 1^{st} term of the A.P. is 3]

Now, $a_1 = 3 + d = 3 + 4 = 7,$

$a_5 = 3 + 5d = 3 + (5)(4) = 23$ [$\because$ a_5 is the 6^{th} term]

and $a_6 = 3 + 6d = 3 + (6)(4) = 27.$ [$\because$ a_6 is the 7^{th} term]

$\therefore \quad a_6 - a_5 = 27 - 23 = 4$ and $a_1 + a_6 = 7 + 27 = 34.$

S6. (i) Let the mean be M_1, M_2, M_3

$\Rightarrow \quad 2, M_1, M_2, M_3, 10$ are in A.P.

$\Rightarrow \quad M_1 = 2 + d,$

$M_2 = 2 + 2d,$

$M_3 = 2 + 3d, \quad 10 = 2 + 4d$

$\Rightarrow \quad 4d = 8 \quad \Rightarrow \quad d = 2$

$\Rightarrow \quad M_1 = 4, \quad M_2 = 6, \quad M_3 = 8.$

(ii) Let the mean be M_1, M_2, M_3, M_4.

$\Rightarrow \quad 1, M_1, M_2, M_3, M_4, 16$ are in A.P.

$\Rightarrow \quad M_1 = 1 + d,$

$M_2 = 1 + 2d,$

$M_3 = 1 + 3d$

$\Rightarrow \quad M_4 = 1 + 4d$

$\Rightarrow \quad 16 = 1 + 5d \quad \Rightarrow \quad 5d = 15$

$\Rightarrow \quad d = 3$

$\Rightarrow \quad M_1 = 4, \quad M_2 = 7, \quad M_3 = 10, \quad M_4 = 13.$

(iii) Let M_1, M_2, M_3, M_4, M_5, be the means

$\Rightarrow$ 3, M_1, M_2, M_3, M_4, M_5, 27 are in A.P.

$\Rightarrow \quad M_1 = 3 + d,$

$M_2 = 3 + 2d,$

$M_3 = 3 + 3d,$

$M_4 = 3 + 4d$

$M_5 = 3 + 5d, \quad 27 = 3 + 6d$

$\Rightarrow \quad 6d = 24 \quad \Rightarrow \quad d = 4$

$\Rightarrow \quad M_1 = 7, \quad M_2 = 11, \quad M_3 = 15, \quad M_4 = 19, \quad M_5 = 23.$

S7. Let a be the first term and d be the common difference of given A.P.

A.M. of t_p and t_q = A.M. of t_r and t_s.

$$\Rightarrow \quad \frac{1}{2}(t_p + t_q) = \frac{1}{2}(t_r + t_s)$$

$$\Rightarrow \quad t_p + t_q = t_r + t_s$$

$$\Rightarrow \quad [a + (p - 1)d + [a + (q - 1)d] = [a + (r - 1)d] + [a + (s - 1)d]$$

$$\Rightarrow \quad (p + q - 2)d = (r + s - 2)d$$

$$\Rightarrow \quad p + q = r + s.$$

S8. Let A_1, A_2,...A_{13} be the 13 A.M.s. between 52 and 17.

Then, 52, A_1, A_2, ..., A_{13}, 17 are in A.P.

Let d be the common difference of this A.P.

1^{st} term = a = 52 and 15^{th} term = t_{15} = 17.

Now, $\quad t_{15} = 17 \quad \Rightarrow \quad a + 14d = 17 \quad \Rightarrow \quad 52 + 14d = 17$

$$\Rightarrow \quad d = \frac{17 - 52}{14} = -\frac{5}{2}$$

Clearly A_2, A_4, A_8, A_{10} and A_{12} are the A.M.s. with integral values.

S9. Let a be the first term and d be the common difference of given A.P.

Let m A.M.s. x_1, x_2,.....x_m be inserted between 5 and 181.

Then 5, x_1, x_2,......, x_n, 181 form an A.P.

This A.P. has first term, $a = 5$, $(m + 2)^{th}$ term = $t_{m+2} = 181$ and 12^{th} term = $t_{12} = 93$.

[$\because$ 93 is the 11^{th} A.M.]

Let d be the common difference of this A.P.

Then, $t_{12} = 93 \Rightarrow 5 + 11\,d = 93 \Rightarrow d = 8.$

And $t_{m+2} = 181 \Rightarrow 5 + (m + 2 - 1)d = 181$

$\Rightarrow 5 + 8(m + 1) = 181 \Rightarrow m = 21.$

S10. Let $x_1, x_2, \ldots x_n$ be n A.M's between 3 and 17.

Then, 3, $x_1, x_2, \ldots, x_n$, 17 are in A.P. Let d be the common difference of this A.P.

Then, $x_n = 17 - d$ and $x_1 = 3 + d.$

Now, $\frac{x_n}{x_1} = \frac{3}{1} \Rightarrow \frac{17 - d}{3 + d} = \frac{3}{1} \Rightarrow 17 - d = 3\,(3 + d) \Rightarrow d = 2.$

Now, $t_{n+2} = 17$

$\Rightarrow 3 + (n + 1)d = 17 \Rightarrow 3 + (n + 1)\,(2) = 1 \Rightarrow n = 6.$

S11. Let A_1, A_2, A_3, A_4, A_5 and A_6 be six numbers between 3 and 24 such that 3, $A_1, A_2, A_3, \ldots, A_6$, 24 are in A.P.

Now let d be the common difference

Here, $T_8 = 24$

$a = 3$

or $a + (8 - 1)\,d = 24$

or $3 + 7d = 24$

or $7d = 21$

$\Rightarrow d = 3$

Thus, $A_1 = a + d = 3 + 3 = 6$

$A_2 = a + 2d = 3 + 2 \times 3 = 9$

$A_3 = a + 3d = 3 + 3 \times 3 = 12$

$A_4 = a + 4d = 3 + 4 \times 3 = 15$

$A_5 = a + 5d = 3 + 5 \times 3 = 18$

$A_6 = a + 6d = 3 + 6 \times 3 = 21$

Six numbers between 3 and 24 are 6, 9, 12, 15, 18 and 21.

S12. Let a be the first term and d be the common difference of given A.P.

$a = 4$ Let the last term be 22

$\therefore$ $l = 22$ We have to insert 5 AM's.

$\therefore$ The AP is of seven terms (5 terms + first term + last term)

$n = 7$

$\therefore$ $l = a + (n-1)d \Rightarrow 22 = 4 + (7-1)d \Rightarrow 18 = 6d \Rightarrow d = 3$

$\therefore$ 1^{st} AM $= a + d = 4 + 3 = 7$

$\therefore$ 2^{nd} AM $= a + 2d = 4 + (2)(3) = 10$

3^{rd} AM $= a + 3d = 4 + (3)(3) = 13$

4^{th} AM $= a + 4d = 4 + (4)(3) = 16$

5^{th} AM $= a + 5d = 4 + (5)(3) = 19.$

S13. Let the m numbers be

$$A_1, A_2, A_3, \ldots, A_m$$

Therefore 1, $A_1, A_2, A_3, \ldots, A_m$, 31 are in A.P.

$$a = 1$$

$$T_{m+2} = 31$$

$\Rightarrow$ $a + (m + 2 - 1)\,d = 31$

or $1 + (m + 1)\,d = 31$

or $d = \dfrac{30}{m+1}$

Thus, $A_7 = a + 7d$

$$= 1 + 7\left(\frac{30}{m+1}\right)$$

$$= \frac{m+1+210}{m+1}$$

$$= \frac{m+211}{m+1}$$

Also, $A_{m-1} = a + (m-1)\,d$

$$= 1 + (m-1)\frac{30}{m+1}$$

$$= \frac{m+1+30m-30}{m+1}$$

$$= \frac{31m-29}{m+1}$$

Given, $\dfrac{A_7}{A_{m-1}} = \dfrac{5}{9}$

or $\dfrac{\dfrac{m+211}{m+1}}{\dfrac{31m-29}{m+1}} = \dfrac{5}{9}$

or $$\frac{m+211}{31m-29} = \frac{5}{9}$$

or $$9m + 1899 = 155m - 145$$

or $$9m - 155m = -145 - 1899$$

or $$-146m = -2044$$

or $$m = \frac{2044}{146} = 14$$

Hence, $$m = 14.$$

S14. Let A be the single A.M. between a and b, then $A = \frac{a+b}{2}$

Let $A_1, A_2, \ldots A_n$ be the n A.Ms. between a and b, then $A = \frac{a+b}{2}$

Let d be the common difference of A.P. Here,

$$T_{n+2} = b$$

$\Rightarrow$ $a + (n + 2 - 1)d = b$ $\quad [\because \ T_n = a + (n-1)d]$

$\Rightarrow$ $a + (n + 1)d = b$ $\Rightarrow$ $d = \frac{b-a}{n+1}$ ... (i)

$$A_1 = T_2 = a + (2-1)d = a + d = a + \frac{b-a}{n+1} \quad \text{... (ii)}$$

$$A_2 = T_3 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$

....

....

$$A_n = T_{n+1} = a + nd = a + n\left(\frac{b-a}{n+1}\right) \quad \text{... (iii)}$$

Sum of n A.Ms. $= A_1 + A_2 + A_3 + \ldots + A_n$

$$= \frac{n}{2}(A_1 + A_n) \quad \left[\because \ S_n = \frac{n}{2}(a+l)\right]$$

$$= \frac{n}{2}\left[a + \frac{b-a}{n+1} + a + n\left(\frac{b-a}{n+1}\right)\right] \quad \text{[From (ii) and (iii)]}$$

$$= \frac{n}{2}\left[a + \frac{b-a}{n+1} + a + n\left(\frac{b-a}{n+1}\right)\right]$$

$$= \frac{n}{2}\left[2a + (n+1)\left(\frac{b-a}{n+1}\right)\right]$$

$$= \frac{n}{2}[2a + b - a] = \frac{n}{2}[a + b] = n\left(\frac{a+b}{2}\right).$$

$\Rightarrow$ Sum of n A.Ms = nA [From (i)]

Hence, the sum of n A.Ms. between a and b is equal to n times the A.M. between a and b.

S15. Since, $a_1, a_2, ...a_n$ are the n arithmetic means between a and b.

$\Rightarrow$ $a, a_1, a_2, ... , a_n, b$ are in A.P.

Let d be the common difference of this A.P.

b is $(n + 2)^{th}$ term of this A.P.

i.e. $b = a + (n + 2 - 1)d \Rightarrow d = \left(\frac{b-a}{n+1}\right)$

Now, $2\sum_{i=1}^{n} a_i = 2(a_1 + a_2 + ... + a_n)$

$$= 2\left[\frac{n}{2}\{2a_1 + (n-1)d\}\right] = n\{2(a + d) + (n + 1)d\}$$

$\left[\because\ a_1, a_2, ..., a_n \text{ is an A.P. of } n \text{ terms having first term } a_1 = a + d \text{ and common difference } d = \left(\frac{b-a}{n+1}\right)\right]$

$$= n\left\{2\left(a + \frac{b-a}{n+1}\right) + (n-1)\left(\frac{b-a}{n+1}\right)\right\}$$

$$= n\left\{\frac{2na + 2b + (n-1)b - (n-1)a}{(n+1)}\right\}$$

$$= n\left\{\frac{(n+1)a + (n+1)b}{(n+1)}\right\} = n(a + b).$$

S16. Let a be the first term and d be the common difference of the given A.P. Then,

a = 20 = first term,

l = 80 = last term

No. of A.M's = n

$\therefore$ $N = n + 2$

Let common difference = d,

$$l = a + (N - 1)d$$

$$80 = 20 + (n + 2 - 1)d$$

$\Rightarrow$ $60 = (n + 1)\,d$

$$d = \frac{60}{n+1} \text{ ... (i)}$$

first mean $= a + d = 20 + d$, last mean $= a + nd = 20 + nd$

Ratio = 1 : 3, $\frac{20+d}{20+nd} = \frac{1}{3}$, $3(20 + d) = 20 + nd$,

$60 + 3d = 20 + nd$

$40 = nd - 3d$, $40 = d(n - 3)$, $d = \frac{40}{n-3}$ ... (ii)

Equating (i) and (ii), we get

$$\frac{60}{n+1} = \frac{40}{n-3}$$

$\Rightarrow$ $60(n - 3) = 40(n + 1)$,

$\Rightarrow$ $60n - 180 = 40n + 40$

$= 20n = 220$

$$n = \frac{220}{20} = 11$$

a = first term = 2, l = last term = 29, No. of A.M.'s = 11.

Hence, $n = 11$.

S17. (i) We know that A.M. between a and $b = \frac{a+b}{2}$

Hence, A.M. between 6 and 12 $= \frac{6+12}{2} = 9$

(ii) We know that A.M. between a and $b = \frac{a+b}{2}$

Hence, A.M. between 5 and 22 $= \frac{5+22}{2} = 13.5$

(iii) We know that A.M. between a and $b = \frac{a+b}{2}$

$a = (\cos\theta + \sin\theta)^2 = \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$b = (\cos\theta - \sin\theta)^2$

$= \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$

$$AM = \frac{a+b}{2} = \frac{1+2\sin\theta\cos\theta+1-2\sin\theta\cos\theta}{2} = \frac{2}{2} = 1$$

(iv) We know that A.M. between a and b = $\frac{a+b}{2}$

$\because$ $(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$

$\therefore$ A.M. between $(x+y)^2$ and $(x-y)^2$ = $\frac{(x+y)^2+(x-y)^2}{2}$

$\Rightarrow$ $2\cdot\frac{x^2+y^2}{2} = x^2+y^2.$

S18. Total no. of terms in AP = 2 + 8 = 10 (one first terms + one last term + no. of AM's)

$n = 10,\quad l = a + (n-1)d,$

$29 = 2 + (10-1)d,$

$27 = 9d,\quad d = 3$

1^{st} A.M = $a + d = 2 + 3 = 5$

2^{nd} A.M = $a + 2d = 2 + 6 = 8$

3^{rd} A.M = $a + 3d = 2 + 9 = 11$

4^{rd} A.M = $a + 4d = 2 + 12 = 14$

5^{th} A. M = $a + 5d = 2 + 15 = 17$

6^{th} A.M = $a + 6d = 2 + 18 = 20$

7^{th} A.M = $a + 7d = 2 + 21 = 23$

8^{th} A.M = $a + 8d = 2 + 24 = 26$

Sum of 8 A.M's = $\frac{n}{2}[2a+(n-1)d] = \frac{8}{2}[2\times5+(8-1)3] = 4[10+21] = 4\times31 = 124$ ...(i)

A.M between 2 and 29

$$= \frac{2+29}{2} = \frac{31}{2}$$

8 times this A.M. = $8\times\frac{31}{2} = 4\times31 = 124$ ... (ii)

From Eq. (i) and (ii), we get

$\therefore$ Sum of 8 AM's = $8\times$ (A.M. between 2 and 29).

Date: 12/10/2021

Q1. A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs. 5 every month, what amount will he pay in the 30^{th} installment.

Q2. The income of a person is Rs. 3,00,000 in the first year and he receives an increase of Rs. 10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

Q3. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by 1/2 km each succeeding hour. After how many hours will the second car overtake the first car, if both cars go non-stop.

Q4. The gate receipts at the show of "Baghbaan" amounted to Rs. 9,500 on the first night and showed a drop of Rs. 250 every succeeding night. If the operational expenses of the show are Rs. 2,000 a day, find on which night the show ceases to be profitable?

Q5. A person buys National Savings Certificates every year of value exceeding the last years purchase by Rs. 100. After 5 year he find that the total value of the certificate is Rs. 5250. Find the value of the certificates purchased by him:

(i) in the 1^{st} year (ii) in the 9^{th} year.

Q6. The ages of the students of a class form an A.P. whose common difference is 4 months. If the youngest student is 8 years old and the sum of the ages of all students of the class is 168 years, find the number of student in the class.

Q7. A man deposited $ 10000 in bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

Q8. The gate receipt at the show of "Rain-Coat" amounted to Rs. 6,500 on the first night and showed a drop of Rs. 110 every succeeding night. If the operational expenses of the show are Rs. 1000 a day. Find on which night the show ceases to be profitable.

Q9. A man buys a car for Rs. 3,00,000.He pays Rs. 1,50,000 in cash and agrees to pay the balance in yearly installments of Rs. 10,000 plus 10% interest on the unpaid amount, find the total amount he paid for the car.

Q10. A manufacturer of radio – set produced 600 units in the third year and 700 units in the seventh year. Assuming that the production uniformly increases by a fixed number every year, find

(a) the production in the first year (b) the total production in 7 years.

Q11. The sides of a right-angled triangle are in A.P. show that the sides are in the ratio 3 : 4 : 5.

Q12. On the first day strike of physicians in a hospital, the attendance of the O.P.D. was 750 patients. As the strike continued the attendance declined by 50 patients every day. Find from which day of the strike the O.P.D. would have no patient.

Q13. 150 workers were engaged to finish a job in a certain number of days, 4 workers dropped out on the second day, 4 more workers dropped out on the third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.

Q14. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Q15. A farmer buys a used scooter for $12,000. He pays $6000 cash and agrees to pay the balance in annual installements of $500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

Q16. Ram buys a scooter for $22000. He pays 4000 cash and agrees to pay the balance in annual installment of $1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Date: 12/10/2021

S1. Thus he pays Rs. 100, Rs. 105, Rs. 110, ...

Now, $a = 100, \quad d = 5$

To find T_{30}

We know that $T_n = a + (n-1)\,d$

$$T_{30} = 100 + (30-1) \times 5$$
$$= 100 + 145 = 245$$

Hence, he will pay Rs. 245 in the 30th installment.

S2. Here, we have an A.P. with $a = 3{,}00{,}000$, $d = 10{,}000$ and $n = 20$.

Using the sum formula, we get,

$$S_{20} = \frac{20}{2}\,[6{,}00{,}000 + 19 \times 10{,}000]$$
$$= 10\,(7{,}90{,}000) = 79{,}00{,}000.$$

Hence, the person received Rs. 79,00,000 as the total amount at the end of 20 years.

S3. Let the second car overtake the first care after n hours. Then, the two cars travel the same distance in n hours.

Distance travelled by the first car in n hours = 10 n km

Distance travelled by the second car in n hours = sum of n terms of an A.P. with first term 8 and common difference $\frac{1}{2}$.

$$= \frac{n}{2}\left[2 \times 8 + (n-1) \times \frac{1}{2}\right] = \frac{n(n+31)}{4}$$

When the second car overtakes the first car, we have

$$10n = \frac{n(n+31)}{4}$$

$$\Rightarrow \quad 40n = n^2 + 31n \Rightarrow n(n-9) = 0 \Rightarrow n = 9 \qquad [\because n \neq 0]$$

Thus, the second car will overtake the first car in 9 hours.

S4. We have cost of gate receipt on the first night (a) = 9500;

Common difference (d) = – 250

The show will cease to be profitable on the night, when

$$2000 = 9500 + (n - 1)(-250)$$

$\Rightarrow \quad 2000 - 9500 = -250n + 250$

$\Rightarrow \quad 2000 - 9500 - 250 = -250n$

$\Rightarrow \quad -7750 = -250n$

$\Rightarrow \quad n = \dfrac{7750}{250} = 31.$

Hence, on 31[st] night the show will cease to be profitable.

S5. Let a be the value of the certificates purchased by him in the 1[st] year.

Common difference is Rs. 100. Then,

$$S_5 = 5250$$

We know that, $\quad S_5 = \dfrac{5}{2}[2a + (5 - 1)d]$

$\Rightarrow \quad 5250 = \dfrac{5}{2}[2a + 4 \times 100]$

$\Rightarrow \quad \dfrac{5250 \times 2}{5} = 2a + 400$

$\Rightarrow \quad 2100 = 2a + 400$

$\Rightarrow \quad 2a = 2100 - 400$

$\Rightarrow \quad a = \dfrac{1700}{2} = 850$

Hence, the value of certificate in the 1[st] year = Rs. 850

$\therefore \quad T_9 = a + (9 - 1)d$

$\Rightarrow \quad T_9 = 580 + 8 \times 100 = 850 = 1,650$

which is the value of certificates purchased in the 9[th] year.

S6. The age of the youngest student (a) = 8 years.

Common difference of their ages (d) = 4 months = $\dfrac{1}{3}$ year.

Sum of ages of all students (S_n) = 168 years.

Let n be the number of students in the class.

We know that, $\quad S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$\Rightarrow$ $$168 = \frac{n}{2}\left[2 \times 8 + (n-1)\cdot\frac{1}{3}\right]$$

$\Rightarrow$ $$2 \times 168 = n\left[16 + \frac{(n-1)}{3}\right]$$

$\Rightarrow$ $$336 = 16n + \frac{n(n-1)}{3}$$

$\Rightarrow$ $$1,008 = 48n + n^2 - n$$

$\Rightarrow$ $$n^2 + 47n - 1,008 = 0$$

$\Rightarrow$ $$n^2 + 63n - 16n - 1,008 = 0$$

$\Rightarrow$ $$n(n + 63) - 16(n + 63) = 0$$

$\Rightarrow$ $$(n + 63)\ (n - 16) = 0$$

Either $n + 63 = 0$ or $n - 16 = 0$

$\Rightarrow$ $n = -63$ or $n = 16$ [$n = -63$ (not possible)]

Hence, the number of students are 16.

S7. Initial deposit = Rs 10,000

Rate of interest = 5% p.a. S.I.

$$\text{Interest for one year} = \text{Rs.}\left(\frac{10,000 \times 5 \times 1}{100}\right) = \text{Rs. } 500$$

The amount in the account of the man in first, second, third, ..., years are Rs. 10,000, Rs. 10,500, Rs. 11,000, ...

Amount in 15^{th} years since deposit = Rs. $(10,000 + 14 \times 500)$

= Rs. 17,000

Amount after 20 years = Rs. $(10,000 + 20 \times 500)$

= Rs. 20,000.

S8. Gate receipt for the show on the first night = Rs. 6,500

Drop every succeeding night = Rs. –110

The show will cease profit on the night when receipt just = Rs. 1000

$$a = 6500,\ d = -110,\ T_n = 1000$$

$$T_n = a + (n-1)d$$

$$1000 = 6500 + (n-1)(-110)$$

$$110\,n = 6500 - 1000 + 110 = 5610$$

$$n = \frac{5610}{110} = 51$$

On 51^{st} night the show will cease to be profitable. After 51 nights there will be no show.

S9. Amount paid cash = Rs. 1,50,000

Remaining amount to be paid = Rs.(300000 – 150,000) = Rs. 1,50,000

$$\text{I installment} = \text{Rs. } 10{,}000 + (1{,}50{,}000)\frac{10}{100} = \text{Rs. } 25{,}000$$

$$\text{II installment} = \text{Rs. } 10{,}000 + (1{,}40{,}000)\frac{10}{100} = \text{Rs. } 24{,}000$$

$$\text{III installment} = \text{Rs. } 10{,}000 + (1{,}30{,}000)\frac{10}{100} = \text{Rs. } 23{,}000$$

$$\text{IV installment} = \text{Rs. } 10{,}000 + (1{,}40{,}000)\frac{10}{100} = \text{Rs. } 24{,}000$$

$$\text{Number of installments} = \frac{1{,}50{,}000}{10{,}000} = 15$$

a = 25000, 24000, 23000.........

$d = -1000$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{15}{2}[2 \times 25{,}000 + (15-1)(-1000)]$$

$$= \frac{15}{2}[50{,}000 - 14000] = \frac{15}{2}(36000) = 15 \times 18000 = 2{,}70{,}000$$

Total amount paid for car = (Rs. 1,50,000 in cash + Rs. 2,70,000) = Rs. 4,20,000.

S10. Let the production in n^{th} year be a_n

a = production in 1st year, d = common difference

$\therefore$ $T_3 = 600$, $a + 2d = 600$ ... (i)

$a_7 = 700$, $a + 6d = 700$ ... (ii)

Solving (i) and (ii), we get

$d = 25$, $a = 550$ units

(a) 1st year production = a = 550 units

(b) the total production in 7 years.

$$S_n = \frac{n}{2}[2a + (n-1)d], \quad n = 7,$$

$$S_7 = \frac{7}{2}[2 \times 550 + (7-1)25] = \frac{7}{2}(1100 + 6 + 25)$$

$$= \frac{1250 \times 7}{2} = 625 \times 7 = 4375.$$

S11. Let the sides be $a - d$, a, $a + d$. are in AP

$a - d, a, a + d$ (given)

By pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(a + d)^2 = a^2 + (a - d)^2$$

$$a^2 + 2ad + d^2 = a^2 + a^2 - 2ad + d^2$$

$$a^2 = 4ad$$

$$a = 4d$$

Sides are $= a - d = 4d - d = 3d$, $a = 4d$, $a + d = 4d + d = 5d$

The sides are in the ratio 3 : 4 : 5.

S12. First term = a = attendance of patients on first day

$\therefore$ $a = 750$

Common difference $d = -50$ (decline mean negative)

let on the n^{th} day, no. of patients = 0

$\therefore$ $a_n = 0, \quad a_n = a + (n-1)d, \quad 0 = 750 + (n-1)(-50)$

$$50(n-1) = 750$$

$$n - 1 = \frac{750}{50} = 15,$$

$$n = 15 + 1 = 16$$

$\therefore$ on 16^{th} day, there will be no patient.

S13. Suppose the work was completed in n days when the workers started dropping. 4 workers dropped out every day except the first day. Therefore, the total number of workers who worked all the n days, is the sum of n terms of an A.P. with first term 150 and common difference -4, *i.e.*

$$\frac{n}{2}[2 \times 150 + (n-1) \times (-4)] = n(152 - 2n)$$

Had the workers not dropped then the work should have finished in $(n - 8)$ days with 150 workers on each day. Therefore, the total number of workers who would have worked all the n day is $150(n - 8)$.

Therefore, $n(152 - 2n) = 150(n - 8)$

or $n^2 - n - 6000 = 0$

or $(n - 25)(n + 24) = 0$

or $n = 25$

Thus, the work was completed in 25 days.

S14.

$$a = \text{Rs. } 15625$$

$$\text{Depreciation} = 20\% \text{ of } 15625$$

$$= \frac{20}{100} \times 15625 = \text{Rs. } 3125$$

$$T_2 = 15625 - 3125 = \text{Rs. } 12500$$

$$\text{Common difference} = 12500 - 15625 = \text{Rs. } -3125$$

$$n = 5$$

$$T_n = a + (n-1)\,d$$

$$T_5 = 15625 + (5-1)(-3125)$$

$$= 15625 - 12500 = \text{Rs. } 3125$$

Hence, estimated value at the end of 5 years = Rs. 3125.

S15. Given, Total cost of a scooter = \$ 12,000

Paid cash = \$ 6,000

Balance = \$ 6,000

Number of installments @ \$ 500 each = 12

$$\text{Interest on first installment} = \$\left(\frac{6000 \times 12 \times 1}{100}\right) = \$\,720$$

$$\text{First installment} = \$\,(500 + 720) = \$\,1220$$

$$\text{Interest on second installment} = \$\left(\frac{5500 \times 12 \times 1}{100}\right) = \$\,660$$

$$\text{Second installment} = \$\,(500 + 660) = \$\,1160$$

$$\text{Interest on third installment} = \$\left(\frac{5000 \times 12 \times 1}{100}\right) = \$\,600$$

$$\text{Third installment} = \$\,(500 + 600)$$

$$= \$\,1100 \text{ and so on}$$

Total amount paid in installments = \$ (1220 + 1160 + 1100 + ... to 12 terms)

Here, $a = 1220, \quad d = -60, \quad n = 12$

$$S = \frac{12}{2}\,[2 \times 1220 + (12-1)(-60)]$$

$$= 6\,(2440 - 11 \times 60)$$

$$= 6\,(2440 - 660)$$

$$= 6 \times 1780 = 10680$$

$$= \$\,10680$$

$\therefore$ Amount paid by farmer = \$ (6000 + 10680)

= \$ 16680.

S16. Given, Total cost of a scooter = \$ 22000

Paid cash = \$ 4000

Balance = \$ 18000

Number of installments @ \$ 1000 each = 18

$$\text{Interest on first installment} = \$ \left(\frac{18000 \times 10 \times 1}{100}\right) = \$ 1800$$

First installment = \$ (1000 + 1800)

= \$ 2800

$$\text{Interest on second installment} = \$ \left(\frac{17000 \times 10 \times 1}{100}\right) = \$ 1700$$

Second installment = \$ (1000 + 1700)

= \$ 2700

$$\text{Interest on third installment} = \$ \left(\frac{16000 \times 10 \times 1}{100}\right) = \$ 1600$$

Third installment = \$ (1000 + 1600)

= \$ 2600 and so on

Total amount paid in installments = \$ (2800 + 2700 + 2600 + ... to 18 terms)

Here, $a = 2800, \quad d = -100, \quad n = 18$

$$S = \frac{18}{2} [2 \times 2800 + (18 - 1)(-100)]$$

$= 9 (5600 - 1700)$

$= 9 \times 3900 = 35100$

= \$ 35100

Amount paid by Ram = \$ (4000 + 35100)

= \$ 39100.

Date: 12/10/2021

Q1. Find the sum of the $\sqrt{7}+\sqrt{21}+\sqrt{63}+....$ to n terms.

Q2. By using geometric series express $0.\overline{3}$ as a rational number.

Q3. Find sum of the series upto infinite terms: $\left(\sqrt{2}+1\right)+1+\left(\sqrt{2}-1\right)+....\infty$

Q4. If the sum of the series $1+\frac{2}{x}+\frac{4}{x^2}+\frac{8}{x^3}+.....\infty$ is a finite number, then prove that $x > 2$.

Q5. If s is the sum of an infinite G.P., and the first term is a, then prove that the common ratio r is given by $r = \frac{s-a}{s}$.

Q6. If the sum of an infinite geometric series is $\frac{4}{3}$ and its first term is $\frac{3}{4}$, then find common ratio of given geometric series.

Q7. The third of a G.P. is 4 find the product of its first five terms.

Q8. If a, b, c are in G.P., prove that log a, log b, log c are in A.P.

Q9. If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$ then show that x, y, z are in A.P.

Q10. If p, q, r are in A.P. show that the p^{th}, q^{th} and r^{th} term of any G.P. are in G.P.

Q11. The sum of an infinite G.P. is 6. If its first term is 2, find its common ratio.

Q12. Find the 20^{th} and n^{th} terms of the G.P. : $\frac{5}{2},\frac{5}{4},\frac{5}{8},....$

Q13. Find the 12^{th} term of a G.P., whose 8^{th} term is 192 and the common ratio is 2.

Q14. Find the sum to indicate number of terms in the following geometric progression: $1, -a, a^2, -a^3, ..., n$ terms (if $a \neq -1$).

Q15. Find the sum to indicate number of terms in the following geometric progression: $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ..., n$ terms.

Q16. Find the sum to indicate number of term in the following geometric progression:

0.15, 0.015, 0.0015, ..., 20 terms.

Q17. For what values of x, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.?

Q18. Which term of the sequence : $\frac{1}{3},\frac{1}{9},\frac{1}{27},...$ is $\frac{1}{19683}$?

Q19. Which term of the sequence : $\sqrt{3}, 3, 3\sqrt{3}, ...$ is 729?

Q20. The 4^{th} term of a G.P. is square of its second term, and the first term is -3. Determine its 7^{th} term.

Q21. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

Q22. Which term of the sequence : $2, 2\sqrt{2}, 4, \ldots$ is 128 ?

Q23. Find the sum to indicate number of terms in the following geometric progression: $x^3, x^5, x^7, \ldots, n$ terms (if $a \neq -1$).

Q24. In a G.P., the 3rd term is 24 and the 6th term is 192. Find the 10 th term.

Q25. Which term of the G.P., 2, 8, 32, ... upto n terms is 131072?

Q26. Find the 10th and n th terms of the G.P., 5, 25, 125, ...

Q27. Evaluate: $$\sum_{k=1}^{11}(2+3^k).$$

Q28. $\frac{1}{x+y}, \frac{1}{2y}, \frac{1}{y+z}$ are the consecutive terms of an A.P., prove that x, y, z are three consecutive terms of a G.P.

Q29. If a, b, c are in A.P. and a, b, d are in G.P., show that a, $a - b$, $d - c$ are in G.P.

Q30. Find the sum of n terms of the series $1 + (1 + x) + (1 + x + x^2) + \ldots\ldots$

Q31. Use geometric series to express $3.5\overline{2}$ as a rational number.

Q32. If $y = x + x^2 + x^3 + \ldots.\infty$, prove that $x = \frac{y}{1+y}$.

Q33. If $x = \left(a + \frac{a}{r} + \frac{a}{r^2} + \ldots\infty\right)$; $y = \left(b - \frac{b}{r} + \frac{b}{r^2} - \ldots\ \infty\right)$; $z = \left(c + \frac{c}{r} + \frac{c}{r^2} + \ldots\ \infty\right)$ then prove that $\frac{xy}{z} = \frac{ab}{c}$, $|r| < 1$.

Q34. The sum of n terms of a progression is $(2^n - 1)$. Show that it is a G.P. Find its common ratio.

Q35. Find the sum of the series .15 + .015 + .0015 +... to 8 terms.

Q36. If a, b, c are in G.P., prove that $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in A.P.

Q37. If $a^x = b^y = c^z$ and x, y, z are in G.P., prove that $\log_b a = \log_c b$.

Q38. If a, b, c are in A.P. and x, y, z are in G.P., show that
$x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$

Q39. If x, y, z be the pth, qth and rth terms of G.P., prove that
$x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$

Q40. If x, $2x + 2$, $3x + 3$,........ are in G.P., then find the fourth term of the given geometric progression.

Q41. Find an infinite G.P. whose first term is 1 and each term is the sum of all the terms which follow it.

Q42. In a G.P., the ratio of the sum of the first 3 terms is to that of first 6 terms is 125 : 152. Find the common ratio of the G.P.

Q43. If a, b, c are in A.P. as well as in G.P., then prove that $a = b = c$.

Q44. Four numbers are in G.P. such that the third term is greater than the first by 9 and the second term is greater than the fourth by 18. Find the fourth term.

Q45. If the first term of an infinite geometric series is 1 and its every term is the sum of the next successive terms, then find its fourth term.

Q46. Find K such that

(a) 7, $K + 1$, 1/7 are in G.P. (b) $K + 9$, $K - 6$, 4 are in G.P.

Q47. Prove that if each term of a G.P. be raised to same power, the resulting terms are also in G.P.?

Q48. If $\frac{1}{x+y}, \frac{1}{2y}, \frac{1}{y+z}$ are in A.P. Prove that x, y, z are in G.P.

Q49. The 4th, 7th and 10th terms of a G.P. are a, b and c respectively. Show that $b^2 = ac$.

Q50. Find the 4th term from the end of the given G.P. $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots$, whose last term is $l = 162$.

Q51. Find the G.P. whose

(a) 5th and 8th terms are 80 and 640 respectively.

(b) 4th and 7th terms are 54 and 1458 respectively.

Q52. The sum of n terms of a progression is $\frac{2^n - 1}{3}$. Show that it is a G.P. and find its common ratio.

Q53. If x, y, z be respectively p^{th}, q^{th} and r^{th} terms of a G.P., prove that $(q - r) \log x + (r - p) \log y + (p - q) \log z = 0$.

Q54. If a, b, c, d are in G.P., prove that the following numbers are also in G.P.

(a) $a + b$, $b + c$, $c + d$ (b) $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$

Q55. Which term of the G.P.

(a) 3, 6, 12, ... is 1536? (b) 5, 10, 20, ... is 10240?

Q56. Find three numbers in G.P. whose sum is 7 and product is 8.

Q57. If a, b, c, d are in G.P. Prove that

(a) $a^2b^2c^2\left[\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right] = a^3 + b^3 + c^3$ (b) $(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$

Q58. Three numbers are in A.P. and their sum is 15. If 1, 3, 9 be added to them respectively, they form a G.P. Find the numbers.

Q59. The product of three number in G.P. is 216 and the sum of the numbers, taken in pairs is 156. Find the numbers.

Q60. The sum of three numbers in G.P. is 56. If 1, 7, 21 are subtracted from the numbers respectively, the resulting numbers from an A.P. Find the G.P.

Q61. The product of the three terms, in G.P., is 1000. If 6 and 7 are added to second and third terms respectively, the terms form an A.P., find the G.P.

Q62. The sum of three numbers, in A.P. is 21. If 2nd is reduced by one and 3rd is increased by one, the resulting, numbers form a G.P. find the numbers.

Q63. (i) How many terms of series 2 + 6 + 18 + ... must be taken to make the sum 728?

(ii) How many terms of the series 1 + 2 + 4 + ... must be taken to make the sum 255?

Q64. Find the sum of the products of the corresponding terms of the sequence 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Q65. Find the sum to n terms of the sequence 8, 88, 888, 8888, ...

Q66. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z respectively, prove that x, y, z are in G.P.

Q67. Find a G.P. for which sum of the first two terms is – 4 and the fifth term is 4 times the third term.

Q68. Given a G.P. with $a = 729$ and 7^{th} term 64, determine S_7.

Q69. How many terms of the G.P. 3, 3^2, 3^3, ..., are needed to give the sum 120?

Q70. If $(m + n)^{th}$ and $(m - n)^{th}$ terms of a G.P. are p and q respectively, show that its m^{th} term is $\sqrt{pq}$.

Q71. If a, b, c are in A.P., and b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P., prove that a, c, e are in G.P.

Q72. Find the sum of n terms of the series whose n^{th} term is $(2^n + 3n)$.

O73. Find sum of the series .7 + .77 + .777 + ... to n terms.

O74. Find sum of the series 3 + 33 + 333 + ... to n terms.

Q75. The 4^{th} and 7^{th} terms of a G.P. are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of the G.P.

Q76. The sum of an infinite G.P. is 15 and the sum of the squares of these terms is 45. Find the series.

Q77. The sum of first two terms of an infinite G.P. 15 and each term is equal to the sum of all the terms following it. Find the G.P.

Q78. Prove that:

(i) $9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} \cdot ... = 3$ (ii) $6^{1/2} \cdot 6^{1/4} \cdot 6^{1/8} \cdot ... = 6$

Q79. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is – 1. FInd the common ratio and the terms.

Q80. Find the sum of first n terms and the sum of first 5 terms of the geometric series $1 + \frac{2}{3} + \frac{4}{9} + ...$

Q81. How many terms of the G.P., $3, \frac{3}{2}, \frac{3}{4}$ are needed to give the sum $\frac{3069}{512}$?

Q82. Find the sum of the following series upto n terms: 0.6 + 0.66 + 0.666 + ...

Q83. Find the sum of the following series upto n terms: 5 + 55 + 555 + ...

Q84. If $\frac{a + bx}{a - bx} = \frac{b + cx}{b - cx} = \frac{c + dx}{c - dx}$, $(x \neq 0)$ then show that a, b, c and d are in G.P.

Q85. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Q86. The sum of some terms of a G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number terms.

Q87. If f is a function satisfying $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, find the value of n.

Q88. If a, b, c and d are in G.P. show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

Q89. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $1/r^n$.

Q90. If the first and the n^{th} terms of a G.P. are a and b, respectively, and if P is the product of first n terms, Prove that $P^2 = (ab)^n$.

Q91. Show that the products of the corresponding terms of the sequence $a, ar, ar^2, ..., ar^{n-1}$ and $A, AR, AR^2, ..., AR^{n-1}$ form a G.P. and find the common ratio.

Q92. If p, q, r are in G.P. and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ also in A.P.

Q93. If p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P. are in G.P. then show that $(p – q)$, $(q – r)$, $(r – s)$ are also in G.P.

Q94. Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.

Q95. If $S_1, S_2, S_3, ... S_n$ are the sums of infinite G.P.'s whose first terms are 1, 2, 3, ... n and common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ... \frac{1}{n+1}$ respectively. Prove that $S_1 + S_2 + S_3 + ... + S_n = \frac{1}{2}n(n + 3)$

Q96. If S_1, S_2, and S_3 be respectively the sums of n, $2n$ and $3n$ terms of a G.P. prove that

(i) $S_1^2 + S_2^2 = S_1(S_2 + S_3)$ (ii) $S_1(S_3 – S_2) = (S_2 – S_1)^2$

Q97. The p^{th}, q^{th} and r^{th} terms of an A.P. as well as those of G.P. are a, b, c respectively, prove that $a^{b-c}. b^{c-a}.c^{a-b} = 1$

Q98. Three numbers are in A.P., and their sum is 18. If the first two numbers are increased by 4 each and the third is increased by 36, the resulting numbers form a G.P. Find the numbers.

Q99. If a, b, c, d are in G.P., prove that the following numbers are also in G.P.

(a) $a^n + b^n, b^n + c^n, c^n + d^n$ (b) $a^2 + b^2 + c^2, ab + bc + cd, b^2 + c^2 + d^2$

Q100 If a, b, c, d are in G.P. Prove that

(a) $a(b^2 + c^2) = c(a^2 + b^2)$ (b) $\frac{1}{a^2 - b^2} + \frac{1}{b^2} = \frac{1}{b^2 - c^2}$

Q101 Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.

Q102 The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, common ratio and the sum to n terms of the G.P.

Q103 The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and terms.

Q104 Find three numbers in G.P. whose sum is 7 and sum of whose squares is 21.

Date: 12/10/2021

S1. The given series is Geometric series with term $a = \sqrt{7}$ and common ratio $r = \dfrac{\sqrt{21}}{\sqrt{7}} = \sqrt{3} > 1$..

$$\therefore \quad S_n = \frac{a(r^n - 1)}{(r-1)} = \frac{\sqrt{7}\{(\sqrt{3})^n - 1\}}{(\sqrt{3}-1)} = \frac{\sqrt{7}(3^{n/2} - 1)}{(\sqrt{3}-1)} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{\sqrt{7}}{2}(\sqrt{3}+1)(3^{n/2} - 1).$$

S2. $0.\overline{3} = 0.333 \ldots\ldots$

$= 0.3 + 0.03 + 0.003 + \ldots.\infty$ $\left[S_\infty = \dfrac{a}{1-r}\right]$

[This is an infinite geometric series with $a = 0.3$ and $r = 0.1$]

$$= \frac{0.3}{1-0.1} = \frac{0.3}{0.9} = \frac{3}{9} = \frac{1}{3}.$$

S3. The given series is a geometric series with first term $a = \sqrt{2}+1$ and common ratio $(\sqrt{2}-1)$,

$$r = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1. \qquad \left[S_\infty = \frac{a}{1-r}\right]$$

$$\therefore \quad S_\infty = \frac{a}{1-r} = \frac{\sqrt{2}+1}{1-(\sqrt{2}-1)} = \frac{\sqrt{2}+1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{4+3\sqrt{2}}{2}.$$

S4. The terms in the given series form an infinite G.P. with first term

$a = 1$ and common ratio $r = \dfrac{2}{x}$

Since the sum of the infinite G.P. is a finite number, so the common ratio

$$|r| < 1 \text{ i.e., } \left|\frac{2}{x}\right| < 1 \quad i.e., \ |x| > 2 \quad i.e., \ x > 2.$$

S5. We have : $s = \dfrac{a}{1-r}$ $\left[\begin{array}{l}\text{sum of an infinite G.P. with first term } a \\ \text{and common ratio } r\end{array}\right]$

$$\Rightarrow \quad 1 - r = \frac{a}{s} \Rightarrow r = 1 - \frac{a}{s} = \frac{s-a}{s} \qquad \left[S_\infty = \frac{a}{1-r}\right]$$

Hence, the common ratio is given by $r = \dfrac{s-a}{s}$.

S6. Let first term $a = \dfrac{3}{4}$. Let the common ratio be r.

Then, sum to infinity, $S_\infty = \dfrac{a}{1-r}$

$$\Rightarrow \qquad \frac{4}{3} = \frac{\left(\frac{3}{4}\right)}{1-r}$$

$$\Rightarrow \qquad 1 - r = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16} \quad \Rightarrow \quad r = 1 - \frac{9}{16} = \frac{7}{16}$$

Thus, the common ratio is $\dfrac{7}{16}$.

S7. Let a be the first term and r be the common ratio of G.P.

$\therefore \quad t_1 = a, \quad t_2 = ar, \quad t_3 = ar^2, \quad t_4 = ar^3, \quad t_5 = ar^4$

Here, $t_3 = 4 \quad \Rightarrow \quad ar^2 = 4$

Now, $\qquad t_1 \cdot t_2 \cdot t_3 \cdot t_4 \cdot t_5 = a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$

$$= a^5 \cdot a^{10} = (ar^2)^5 = 1024.$$

S8. a, b, c are in G.P. $\quad \Rightarrow \quad b^2 = ac$

Taking log both sides, we get

$$\Rightarrow \qquad 2 \log b = \log a + \log c$$

$\Rightarrow \quad \log a$, $\log b$, $\log c$, are in A.P.

S9. a, b, c are in G.P. $\quad \Rightarrow \quad b^2 = ac$ ... (i)

Now, $\qquad a^{1/x} = b^{1/y} = c^{1/z} = k$

From (i), $\qquad (k^y)^2 = k^x \cdot k^z = k^{2y} = k^{x+z}$

Comparing exponents of each terms in k, we get

$$\Rightarrow \qquad 2y = x + z \qquad [k > 0]$$

$\Rightarrow \quad x$, y, z are in A.P.

S10. Let A be the first term and R be the common ratio of G.P.

p, q, r are in A.P. $\quad \Rightarrow \quad 2q = p + r$

$$t_p = Ar^{p-1},\ t_q = Ar^{q-1},\ t_r = AR^{r-1}$$

Now, $(t_q)^2 = [AR^{q-1}]^2 = A^2R^{2q-2} = A^2R^{p+r-2}$

$= AR^{p-1} \cdot AR^{r-1} = t_p \cdot t_r$

hence t_p, t_q, t_r are in G.P.

S11. Let a be the first term and r be the common ratio of G.P.

Here, $S = 6, a = 2$

$$S = \frac{a}{1-r} \quad \Rightarrow \quad 6 = \frac{2}{1-r}$$

$\Rightarrow$ $6(1 - r) = 2 \quad \Rightarrow \quad r = \frac{2}{3}$

S12. Let a be the first term and r be the common ratio of given G.P.

G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Here, $a = \frac{5}{2}$

$r = \frac{5/4}{5/2} = \frac{1}{2}$

$T_n = a\,r^{n-1}$

Thus, $T_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19}$

and $T_n = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1}$.

S13. Let for the G.P. : First term = a

and Common ratio = $r = 2$

Given, $T_8 = 192$

To find : T_{12}

Now, $T_8 = 192$

$\Rightarrow$ $a\,r^{8-1} = 192$

or $a\,(2)^7 = 192$

or $a = \frac{192}{128} = \frac{3}{2}$

Therefore, $T_{12} = a\,r^{11} = \frac{3}{2}(2)^{11}$

$= 3\,(2^{10})$

$= 3 \times 1024 = 3072.$

S14. Let first term = a and common ratio = r for the given G.P.

G.P. is $1, -a, a^2, -a^3, \ldots$

Now, first term $A = 1$

$$r = -a$$

$$S_n = \frac{A(1-r^n)}{1-r}$$

$$= \frac{1[1-(-a)^n]}{1-(-a)}$$

$$= \frac{1-(-a)^n}{1+a}.$$

S15. Let first term = a and common ratio = r for the given G.P.

Here, $a = \sqrt{7}$

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{\frac{21}{7}}$$

$$= \sqrt{3} > 1,$$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$= \frac{\sqrt{7}[(\sqrt{3})^n - 1]}{\sqrt{3}-1}$$

$$= \frac{\sqrt{7}[(\sqrt{3}+1)[(\sqrt{3})^n - 1]]}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{\sqrt{7}[(\sqrt{3}+1)[(\sqrt{3})^n - 1]]}{2}.$$

S16. Let first term = a and common ratio = r for the given G.P.

Given, $a = 0.15$

$$r = \frac{0.015}{0.15} = 0.1, \ < 1$$

$$n = 20$$

$$S_n = \frac{a(1-r^n)}{1-r}, \ r < 1$$

$$S_{20} = \frac{0.15[1-(0.1)^{20}]}{1-0.1}$$

$$= \frac{0.15[1-(0.1)^{20}]}{0.9}$$

$$= \frac{1}{6}[1-(0.1)^{20}].$$

S17. Given, $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

$$x^2 = \left(\frac{-7}{2}\right)\left(\frac{-2}{7}\right)$$

or $$x^2 = 1$$

Hence, $$x = \pm 1.$$

S18. Let first term = a and common ratio = r for the given G.P.

Here $$a = \frac{1}{3}$$

$$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{9} \times \frac{3}{1} = \frac{1}{3}$$

Let $$T_n = \frac{1}{19683}$$

or $$a\,r^{n-1} = \frac{1}{19683}$$

or $$\frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$

or $$\left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9$$

or $$n = 9$$

Hence, $\frac{1}{19683}$ is the 9[th] term of the given G.P.

S19. Let first term = a and common ratio = r for the given G.P.

Here, $$a = \sqrt{3}$$

$$r = \sqrt{3}$$

Let $$T_n = 729$$

$\Rightarrow$ $$a\,r^{n-1} = 729$$

or $$(\sqrt{3})(\sqrt{3})^{n-1} = 729$$

or $$(\sqrt{3})^n = (9)^3$$

or $$3^{n/2} = (3^2)^3 = 3^6$$

or $$\frac{n}{2} = 6$$

or $$n = 12$$

Hence, 729 is the 12[th] term of the given G.P.

S20. Let first term = a and common ratio = r for the given G.P.

Given $T_4 = (T_2)^2$ and $a = -3$

= $ar^3 = (ar)^2$

or $(-3)\,r^3 = (-3)^2 r^2$

or $r = -3$

Thus, $T_7 = ar^6 = (-3)\,(-3)^6$

$= (-3)^7 = -2187.$

S21. Given, $T_5 = p,\quad T_8 = q,\quad T_{11} = s$

Let for the G.P. : First term = a

and Common ratio = r

$$T_5 = a\,r^{5-1} = a\,r^4 = p$$
$$T_8 = a\,r^{8-1} = a\,r^7 = q$$
$$T_{11} = a\,r^{11-1} = a\,r^{10} = s$$

Now, L.H.S. = $q^2 = (a\,r^7)^2 = a^2r^{14}$

R.H.S. = $ps = (a\,r^4)(a\,r^{10}) = a^2r^{14}$

Hence, $q^2 = ps$. **Proved.**

S22. Let first term = a and common ratio = r for the given G.P.

Here, $a = 2$

$r = \sqrt{2}$

Let $T_n = 128$

$\Rightarrow$ $a\,r^{n-1} = 128$

or $2(\sqrt{2})^{n-1} = 128$

or $(\sqrt{2})^{n-1} = 64$

or $2^{\frac{n-1}{2}} = 2^6$

or $\dfrac{n-1}{2} = 6$

or $n = 13$

Hence, 128 is the 13[th] term of the given G.P.

S23. Let first term = a and common ratio = r for the given G.P.

G.P. is $x^3, x^5, x^7, \ldots$

Here, $a = x^3$

$$r = \frac{x^5}{x^3} = x^2$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{x^3[1-(x^2)^n]}{1-x^2}$$

$$= \frac{x^3(1-x^{2n})}{1-x^2}.$$

S24. Here, $a_3 = ar^2 = 24$ ... (i)

and $a_6 = ar^5 = 192$ ... (ii)

Dividing Eq. (ii) by Eq. (i), we get $r = 2$.

Substituting $r = 2$ in Eq. (i), we get $a = 6$.

Hence, $a_{10} = 6(2)^9 = 3072$.

S25. Let 131072 be the n^{th} term of the given G.P. Here, $a = 2$ and $r = 4$.

Therefore, $131072 = a_n = 2(4)^{n-1}$ or $65536 = 4^{n-1}$

This gives $4^8 = 4^{n-1}$

So that $n - 1 = 8$, *i.e.*, $n = 9$. Hence, 131072 is the 9^{th} term of the G.P.

S26. Here, $a = 5$ and $r = 5$.

Thus, $a_{10} = 5(5)^{10-1} = 5(5)^9 = 5^{10}$

and $a_n = a\,r^{n-1} = 5(5)^{n-1} = 5^n$.

S27. Here, $\sum_{k=1}^{11}(2+3^k) = (2 + 3^1) + (2 + 3^2) + (2 + 3^3) + ... + (2 + 3^{11})$

$$= 2 \times 11 + (3^1 + 3^2 + 3^3 + ... + 3^{11})$$

$$= 22 + \frac{3(3^{11}-1)}{3-1} \qquad \left[S = \frac{a(r^n-1)}{r-1}, \quad r > 1\right]$$

$$= 22 + \frac{3}{2}(3^{11}-1).$$

S28. $\frac{1}{x+y}, \frac{1}{2y}, \frac{1}{y+z}$ are three consecutive terms of an A.P.

$\Rightarrow$ $2\left(\frac{1}{2y}\right) = \frac{1}{x+y} + \frac{1}{y+z} \Rightarrow (x+y)(y+z) = y(y+z+x+y)$

$\Rightarrow$ $xy + xz + y^2 + yz = 2y^2 + xy + yz \Rightarrow y^2 = xz$

$\Rightarrow$ x, y, z are in G.P.

Hence, x, y, z are three consecutive terms of a G.P.

S29. a, b, c are in A.P. $\Rightarrow$ $b = \left(\frac{a+c}{2}\right)$ ... (i)

a, b, d are in G.P. $\Rightarrow$ $b^2 = ad$ .. (ii)

Now, $(a - b)^2 = a^2 + b^2 - 2ab$

$= a^2 + ad - 2a\left(\frac{a+c}{2}\right)$ [Using (i) and (ii)]

$= a^2 + ad - a(a + c) = ad - ac = a(d - c)$

$\therefore$ $(a - b)^2 = a(d - c)$

$\therefore$ a, $a - b$, $d - c$ are in G.P.

S30. $S_n = 1 + (1 + x) + (1 + x + x^2) + ...$ to n terms

$\Rightarrow$ $(1 - x)\, S_n = (1 - x) + (1 - x^2) + (1 - x^3) + ... + (1 - x^n)$ $\left[S_n = \frac{a(1-r^n)}{(1-r)}\right]$

$= n - (x + x^2 + x^3 + ... + x^n) = n - \frac{x(1-x^n)}{(1-x)}$

$\Rightarrow$ $S_n = \frac{1}{(1-x)}\left\{n - \frac{x(1-x^n)}{(1-x)}\right\}$.

S31. $3.5\bar{2} = 3.52222$ ....

$= 3.5 + 0.02 + 0.002 + 0.0002 + ... \infty$ $\left[S_\infty = \frac{a}{1-r}\right]$

$= \frac{35}{10} + \left\{\frac{0.02}{1-0.1}\right\} = \frac{35}{10} + \frac{0.02}{0.9} = \frac{35}{10} + \frac{2}{90}$.

$= \frac{315+2}{90} = \frac{317}{90} = 3\frac{47}{90}$.

$\left[\because\ 0.02 + 0.002 + 0.0002 + ...\infty\right.$ is infinite G.P. with first term a = 0.02 and common ratio $\left. r = 0.1\right]$

S32. Give, $y = x + x^2 + x^3 + ...\ \infty$

$= x(1 + x + x^2 + ...\ \infty)$ $\left[S_\infty = \frac{a}{1-r}\right]$

$= x.\frac{1}{(1-x)} = \frac{x}{1-x}$

$\Rightarrow$ $y - yx = x \;\;\Rightarrow\;\; y = x + xy \;\;\Rightarrow\;\; x = \frac{y}{1+y}$.

S33. On summing each infinite geometric series, we get

$$x = \frac{a}{\left(1-\frac{1}{r}\right)} = \frac{ar}{(r-1)}; \; y = \frac{b}{1-\left(-\frac{1}{r}\right)} = \frac{br}{(r+1)};$$

$$z = \frac{c}{1-\left(\frac{1}{r^2}\right)} = \frac{cr^2}{(r^2-1)};$$

$\therefore$ $$\frac{xy}{z} = \left(\frac{ar}{r-1}\right)\left(\frac{br}{r+1}\right)\left(\frac{r^2-1}{cr^2}\right) = \frac{ab}{c} \qquad \left[S_\infty = \frac{a}{1-r}\right]$$

S34. We have : $S_n = 2^n - 1$

$\therefore$ $S_{n-1} = 2^{n-1} - 1$

And so, $t_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1) = 2^n - 2^{n-1}$

$= 2^{n-1}(2 - 1) = 2^{n-1}.$

$\therefore$ $t_{n-1} = 2^{(n-1)-1} = 2^{n-2}.$

Now, $\frac{t_n}{t_{n-1}} = \frac{2^{n-1}}{2^{n-2}} = 2$, which is independent of n.

Hence, the given progression is a G.P. with common ratio 2.

S35. The given series is a Geometric Series with first term, $a = 0.15$ and common ratio 0.1.

$$r = \frac{0.015}{0.15} = \frac{1}{10} = 0.1 < 1$$

$\therefore$ $$S_n = \frac{a(1-r^n)}{(1-r)}$$

$\Rightarrow$ $$S_8 = \frac{(0.15)\{1-(0.1)^8\}}{(1-0.1)} = \left(\frac{0.15}{0.9}\right)\{1-(0.10)^8\}$$

$$= \frac{15}{90}\left(1-\frac{1}{10^8}\right) = \frac{1}{6}\left(1-\frac{1}{10^8}\right).$$

S36. a, b, c are in G.P $\Rightarrow$ $b^2 = ac$ … (i)

Let $\log_a m = x, \quad \log_b m = y, \quad \log_c m = z$

$\Rightarrow$ $a^x = m, \quad b^y = m, \quad c^z = m$

$\Rightarrow$ $a = m^{1/x}, \quad b = m^{1/y}, \quad c = m^{1/z}$

Now, from (i)

$$(m^{1/y})^2 = m^{1/x} \cdot m^{1/z} \quad \Rightarrow \quad m^{2/y} = m^{\frac{1}{x}+\frac{1}{z}}$$

$\Rightarrow$ $$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$\Rightarrow$ $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. Hence the Problem

S37. x, y, z are in G.P. $\Rightarrow$ $y^2 = xz$ ... (i)

Now, $$a^x = b^y = c^z = k \text{ (say)}$$

$\Rightarrow$ $$\log_a k = x, \quad \log_b k = y, \quad \log_c k = z$$

Form (i),

$$(\log_b k)^2 = \log_a k \,.\, \log_c k$$

$\Rightarrow$ $$\frac{\log_b k}{\log_a k} = \frac{\log_c k}{\log_b k} \quad \Rightarrow \quad \log_b a = \log_c b.$$

S38. a, b, c are in A.P. $\Rightarrow$ $2b = a + c$ ... (i)

x, y, z are in G.P. $\Rightarrow$ $y^2 = xz$ ... (ii)

Now $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = x^{b-c} \cdot (xz)^{(c-a)/2} \cdot z^{a-b}$ [From (ii)]

$$= x^{b-c+(c-a)/2} \cdot z^{-b+a+(c-a)/2}$$

$$= x^{(a+c-c-a)/2} \cdot z^{(2b-2b)/2}$$ [From (i)]

$$= x^0 \cdot z^0 = 1.$$

S39. Here, $$t_p = x, \quad t_q = y, \quad t_r = z$$

$\Rightarrow$ $$AR^{p-1} = x, \quad AR^{q-1} = y, \quad AR^{r-1} = z$$

Now, $$x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = [AR^{p-1}]^{q-r} \cdot [AR^{q-1}]^{r-p} \cdot [AR^{r-1}]^{p-q}$$

$$= A^{(q-r+r-p+p-q)} \cdot R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$

$$= A^0 \cdot R^0 = (AR)^0 = 1.$$

S40. Let r be the common ratio of the G.P. x, $2x + 2$, $3x + 3$, ...

Then, $$r = \frac{3x+3}{2x+2} = \frac{3(x+1)}{2(x+1)} = \frac{3}{2}$$

Now, $$2x + 2 = (x)\left(\frac{3}{2}\right) \Rightarrow \frac{x}{2} = -2 \quad \Rightarrow x = -4.$$

$\therefore$ The fourth term of the G.P. is

$$t_4 = (x)(r)^3 = (-4)\left(\frac{3}{2}\right)^3 = \frac{-27}{2} = -13.5.$$

S41. Here, $a = 1$

$\therefore$ the G.P. is $1, r, r^2, \ldots$

Now, $1 = r + r^2 + r^3 + \ldots = \dfrac{r}{1-r}$

$\Rightarrow$ $1 - r = r \Rightarrow r = \dfrac{1}{2}$

$\therefore$ the G.P. is $1, \dfrac{1}{2}, \dfrac{1}{4}, \ldots$

S42. Let a be the first term and r be the common ratio of the G.P.

Then, $S_3 : S_6 = 125 : 152 \Rightarrow \dfrac{S_3}{S_6} = \dfrac{125}{152}$ $\left[S_n = \dfrac{a(1-r^n)}{(1-r)}\right]$

$\Rightarrow$ $\dfrac{a(r^3-1)}{(r-1)} \times \dfrac{(r-1)}{a(r^6-1)} = \dfrac{125}{152}$

$\Rightarrow$ $\dfrac{(r^3-1)}{(r^3+1)\times(r^3-1)} = \dfrac{125}{152}$

$\Rightarrow$ $r^3 + 1 = \dfrac{152}{125}$

$\Rightarrow$ $r^3 = \dfrac{27}{125} = \left(\dfrac{3}{5}\right)^3 \Rightarrow r = \dfrac{3}{5}.$

Hence, the common ratio of the G.P. is $\dfrac{3}{5}$.

S43. a, b, c, are in A.P. $b = \dfrac{a+c}{2}$ ... (i)

a, b, c, are in G.P. $b^2 = ac$ ... (ii)

From (i) and (ii), we get:

$$\left(\frac{a+c}{2}\right)^2 = ac \Rightarrow (a+c)^2 - 4ac = 0 \Rightarrow (a-c)^2 = 0 \Rightarrow a = c.$$

Substituting $a = c$ in (i) we get : $b = \dfrac{2a}{2} = a.$

Hence, $a = b = c$.

S44. Let a be the first term and r be the common ratio of the G.P.

Let the four terms of the G.P. be a, ar, ar^2, ar^3.

Then, $t_3 - t_1 = 9$ and $t_2 - t_4 = 18$

$\Rightarrow$ $ar^2 - a = 9$ and $ar - ar^3 = 18$ ... (i)

$\Rightarrow$ $a(r^2 - 1) = 9$ and $(-ar)(r^2 - 1) = 18$ ... (ii)

Dividing Eq. (ii) by (i), we get

$$\therefore \quad \frac{(-ar)(r^2-1)}{a(r^2-1)} = \frac{18}{9} \Rightarrow -r = 2 \quad \Rightarrow r = -2.$$

Putting $r = -2$ in $a(r^2 - 1) = 9$ we get: $a = 3$

$\therefore$ The fourth term = $t_4 = ar^3 = 3(-2)^3 = -24$.

S45. Let the required G.P. be $1, r, r^2, r^3, \ldots \infty$

Then, $1 = (r + r^2 + r^3 + \ldots \infty)$ $\left[S_\infty = \frac{a}{1-r}\right]$

[***Note:*** $|r| < 1$ since it is an infinite G.P. with a finite sum of its terms.]

$$\Rightarrow \quad 1 = \frac{r}{1-r}$$

$$\Rightarrow \quad 1 - r = r \Rightarrow 2r = 1 \Rightarrow r = \frac{1}{2}$$

$\therefore$ Fourth term of the G.P. = $t_4 = ar^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

S46. (a) Since, $7, K + 1, 1/7$ are in G.P. if

$$\frac{K+1}{7} = \frac{1}{7(K+1)}$$

$\Rightarrow$ $(K + 1)^2 = 1$ $\Rightarrow$ $K = 0, -2$

(b) Since, $K + 9, K - 6, 4$ are in G.P. if

$$\frac{K-6}{K+9} = \frac{4}{K-6}$$

$\Rightarrow$ $(K - 6)^2 = 4(K + 9)$

$\Rightarrow$ $K^2 - 12K + 36 = 4K + 36$

$\Rightarrow$ $K^2 - 16K = 0$ $\Rightarrow$ $K = 0, 16$.

S47. Let a be the first term and r be the common ratio of G.P.

Then, the G.P. be a, ar, ar^2, ...

and let each term be raised to power k. $(K \in R)$

Therefore, the new terms are

$$a^k, (ar)^k, (ar^2)^k, ...$$

Clearly, the resulting terms are in G.P. with first term a^k and common ratio r^k.

S48. Since, $\frac{1}{x+y}, \frac{1}{2y}, \frac{1}{y+z}$ are in A.P.

$\Rightarrow \quad 2\left(\frac{1}{2y}\right) = \frac{1}{x+y} + \frac{1}{y+z}$

$\Rightarrow \quad \frac{1}{y} = \frac{y+z+x+y}{(x+y)(y+z)}$

$\Rightarrow \quad (x + y)(y + z) = y(x + 2y + z)$

$\Rightarrow \quad xy + xz + y^2 + yz = xy + 2y^2 + yz$

$\Rightarrow \quad y^2 = xz$

$\Rightarrow$ x, y, z are in G.P.

S49. Let A be the first term and R be the common ratio of G.P.

Here, $\quad t_4 = a, \quad t_7 = b, \quad t_{10} = c$

$\Rightarrow \quad AR^3 = a, \quad AR^6 = b, \quad AR^9 = c$

Now $\quad b^2 = (AR^6)^2 = A^2R^{12}$

$\quad = (AR^3)\ (AR^9) = ac.$

$\therefore$ Hence, $b^2 = ac$.

S50. The last term of the G.P. is $l = 162$.

The common ratio of the G.P is $r = \frac{\left(\frac{2}{9}\right)}{\left(\frac{2}{27}\right)} = 3$.

The m^{th} term from the end of the G.P. = $\frac{1}{r^{m-1}} \times 162 = \frac{162}{3^{m-1}}$

$\therefore$ The 4^{th} term from the end of the G.P. = $\frac{162}{3^{4-1}} = 6$.

S51. (a) Here, $t_5 = 80$ and $t_8 = 640$

$\Rightarrow \quad ar^4 = 80 \quad$ and $\quad ar^7 = 640$

Dividing these equations, we get

$$\frac{1}{r^3} = \frac{1}{8} \quad \Rightarrow \quad r^3 = 8 \quad \Rightarrow \quad r = 2$$

Now, $\quad a = \frac{80}{r^4} = \frac{80}{16} = 5$

Hence, the required G.P. is 5, 10, 20,...

(b) Here, $t_4 = 54$ and $t_7 = 1458$

$\Rightarrow$ $ar^3 = 54,\ ar^6 = 1458$

Dividing these equations, we get

$$\frac{1}{r^3} = \frac{1}{27}$$

$$\Rightarrow \quad r = 3,\ a = \frac{54}{r^3} = \frac{54}{27} = 2$$

Hence, the required G.P. is 2, 6, 18,...

S52.

$$S_n = \frac{2^n - 1}{3} \quad \Rightarrow \quad S_{n-1} = \frac{2^{n-1} - 1}{3}$$

We know that $t_n = S_n - S_{n-1}$

$$= \frac{2^n - 1}{3} - \frac{2^{n-1} - 1}{3} = \frac{2^n - 2^{n-1}}{3}$$

$$\Rightarrow \quad t_{n-1} = \frac{2^{n-1} - 2^{n-2}}{3}$$

$$\frac{t_n}{t_{n-1}} = \frac{\left(\dfrac{2^n - 2^{n-1}}{3}\right)}{\left(\dfrac{2^{n-1} - 2^{n-2}}{3}\right)}$$

$$= \frac{2(2^{n-1} - 2^{n-2})}{(2^{n-1} - 2^{n-2})} = 2$$

As t_n/t_{n-1} is independent of n, we will get a G.P. with $r = 2$.

S53. Let A be the first term and R be the common ratio of G.P.

Here, $t_p = x, \quad t_q = y, \quad t_r = z$

$\Rightarrow$ $AR^{p-1} = x,\ AR^{q-1} = y,\ AR^{r-1} = z$

$\Rightarrow$ $\log x = \log A + (p - 1) \log R,$

$$\log y = \log A + (q - 1) \log R$$

$$\log z = \log A + (r - 1) \log R$$

Now $(q - r) \log x + (r - p) \log y + (p - q) \log z$

$$= (q - r)\,[\log A + (p - 1) \log R] + (r - p)$$
$$[\log A + (q - 1) \log R] + (p - q)\,[\log A + (r - 1) \log R]$$

$= \log A[q - r + r - p + p - q] + \log R\,[(p - 1)(q - r) + (q - 1)$

$(r - p) + (r - 1)\,(p - q)]$

$= (0) \log A + (0) \log R = 0.$

S54. Let a be the first term and r be the common ratio of G.P.

Let, $b = ar$, $c = ar^2$, $d = ar^3$

(a) $$(b + c)^2 = (ar + ar^2)^2 = a^2r^2(1 + r)^2 \quad \text{... (i)}$$

$$(a + b)\,(c + d) = (a + ar)\,(ar^2 + ar^3)$$
$$= a\,(1 + r)ar^2(1 + r)$$
$$= a^2r^2(1 + r)^2 \quad \text{... (ii)}$$

From (i) and (ii)

$$(b + c)^2 = (a + b)\,(c + d)$$

$\Rightarrow$ $a + b$, $b + c$, $c + d$ are in G.P.

(b) $$(b^2 + c^2)^2 = (a^2\,r^2 + a^2r^4)^2 \quad \text{... (i)}$$
$$= a^4r^4(1 + r^2)^2$$
$$(a^2 + b^2)\,(c^2 + d^2) = (a^2 + a^2r^2)\,(a^2r^4 + a^2r^6)$$
$$= a^2(1 + r^2) \cdot a^2r^4\,(1 + r^2)$$
$$= a^4r^4\,(1 + r^2)^2 \quad \text{... (ii)}$$

From (i) and (ii)

$$(b^2 + c^2)^2 = (a^2 + b^2)\,(c^2 + d^2)$$

$\Rightarrow$ $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P.

S55. (a) Here, $a = 3$, $r = 2$, $t_n = 1536$

$t_n = ar^{n-1} \Rightarrow 1536 = 3.2^{n-1}$

$\Rightarrow$ $2^{n-1} = 512$

$\Rightarrow$ $2^{n-1} = 2^9 \Rightarrow n - 1 = 9$

$\Rightarrow$ $n = 10$

Hence, 10th term is 1536.

(b) Here, $a = 5$, $r = 2$, $t_n = 10240$

$t_n = ar^{n-1} \Rightarrow 10240 = 5\,.\,2^{n-1}$

$\Rightarrow$ $2^{n-1} = 2048 = 2^{11}$

$\Rightarrow$ $n - 1 = 11 \Rightarrow n = 12$

Hence, the 12th term is 10240.

S56. Let the numbers be a/r, a, ar.

Now, $\frac{a}{r} \cdot a \cdot ar = 8 \quad \Rightarrow \quad a^3 = 8$

$\Rightarrow \quad a = 2$

and $\frac{a}{r} + a + ar = 7$

$\Rightarrow \quad 2 + 2r + 2r^2 = 7r$

$\Rightarrow \quad 2r^2 - 5r + 2 = 0$

$\Rightarrow \quad (r - 2)(2r - 1) = 0$

$\Rightarrow \quad r = 2, \frac{1}{2}$

For $r = 2$, a = 2, hence G.P be 1,2,4

For $r = \frac{1}{2}, a = 2$ hence G.P be 4,2,1

Hence the numbers are 1, 2, 4 or 4, 2, 1.

S57. Let a be the first term and r be the common ratio of the given G.P.

(a) From L.H.S.

$$a^2b^2c^2\left[\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right] = a^2(a^2r^2)(a^2r^4)\left[\frac{1}{a^3}+\frac{1}{a^3r^3}+\frac{1}{a^3r^6}\right]$$

$$= a^6r^6\left[\frac{1}{a^3}+\frac{1}{a^3r^3}+\frac{1}{a^3r^6}\right]$$

$$= a^3r^6 + a^3r^3 + a^3$$

$$= (ar^2)^3 + (ar)^3 + (a)^3 = c^3 + b^3 + a^3. \text{ R.H.S.}$$

Hence, $a^2b^2c^2\left[\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right] = a^3 + b^3 + c^3$

(b) From L.H.S.

$$(a + 2b + 2c)(a - 2b + 2c) = (a + 2ar + 2ar^2)(a - 2ar + 2ar^2)$$

$$= a^2(1 + 2r + 2r^2)(1 - 2r + 2r^2)$$

$$= a^2[(1 + 2r^2)^2 - (2r)^2]$$

$$= a^2(1 + 4r^4) = a^2 + 4(ar^2)^2 = a^2 + 4c^2. \text{ R.H.S.}$$

Hence, $(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$

S58. Let the number be $a - d$, a, $a + d$.

Now, $(a - d) + a + (a + d) = 15$

$\Rightarrow \quad 3a = 15 \quad \Rightarrow \quad a = 5$

Now, $a - d + 1$, $a + 3$, $a + d + 9$ are in G.P.

$\Rightarrow$ $6 - d$, 8, $14 + d$ are in G.P.

$\Rightarrow$ $(6 - d)(14 + d) = (8)^2$

$\Rightarrow$ $84 - 8d - d^2 = 64$

$\Rightarrow$ $d^2 + 8d - 20 = 0$

$\Rightarrow$ $(d + 10)(d - 2) = 0 \Rightarrow d = 2, -10$

For $a = 5$, $d = 2$, $a - d = 3$, $a = 5$, $a + d = 7$

For $a = 5$, $d = -10$, $a - d = 15$,

$a = 5$, $a + d = -5$

Hence, the numbers are 3, 5, 7 or 15, 5, – 5.

S59. Let the numbers be $\frac{a}{r}, a, ar$

Now, $\frac{a}{r} \cdot a \cdot ar = 216$

$\Rightarrow$ $a = 6$ and $\frac{a}{r} \cdot a + a \cdot ar + \frac{a}{r} \cdot ar = 156$

$\Rightarrow$ $\frac{a^2}{r} + a^2 r + a^2 = 156$

$\Rightarrow$ $a^2[1 + r + r^2] = 156\, r$

$\Rightarrow$ $36\,[1 + r + r^2] = 156\, r$

$\Rightarrow$ $3\,(1 + r + r^2) = 13r$

$\Rightarrow$ $3r^2 - 10r + 3 = 0$

$\Rightarrow$ $(r - 3)(3r - 1) = 0$

$\Rightarrow$ $r = 3, \frac{1}{3}$ For $a = 6$, $r = 3$

hence the G.P be 2,6,18.

For r = 1/3 , a=6 G.P be 18,6,2

Hence the number are 2, 6, 18, or 18, 6, 2.

S60. Let the numbers be a, ar, ar^2.

Now, $a + ar + ar^2 = 56$

$\Rightarrow$ $a(1 + r + r^2) = 56$ … (i)

Now, $a - 1$, $ar - 7$, $ar^2 - 21$ are in A.P.

$\Rightarrow \quad 2(ar - 7) = (a - 1) + ar^2 - 21$

$\Rightarrow \quad 2ar - 14 = a + ar^2 - 22$

$\Rightarrow \quad 3ar + 8 = a + ar + ar^2 = 56$ [From (i)]

$\Rightarrow \quad 3ar = 48 \Rightarrow ar = 16 \Rightarrow a = \frac{16}{r}$

From (i) $\quad \frac{16}{r}(1 + r + r^2) = 56$

$\Rightarrow \quad 2(1 + r + r^2) = 7r$

$\Rightarrow \quad 2r^2 - 5r + 2 = 0$

$\Rightarrow \quad (r - 2)(2r - 1) = 0$

$\Rightarrow \quad r = 2, \frac{1}{2}$

When $\quad r = 2, \quad a = \frac{16}{2} = 8$

When $\quad r = \frac{1}{2}, \quad a = 16.2 = 32$

hence required G.P be 8,16,32 for r=2, a=8 and 32,16,8 for r=1/2, a=32

Hence, the number are 8, 16, 32, or 32, 16, 8.

S61. Let the numbers be $\frac{a}{r}, a, ar$.

Now, $\quad \frac{a}{r} \cdot a \cdot ar = 1000$

$\Rightarrow \quad a^3 = 1000 \Rightarrow a = 10$

Now, $\frac{a}{r}, a + 6, ar + 7$ in A.P.

$\Rightarrow \quad 2(a + 6) = \frac{a}{r} + ar + 7$

$\Rightarrow \quad 2(10 + 6) = \frac{10}{r} + 10r + 7$

$25 = \frac{10}{r} + 10r$

$2r^2 - 5r + 2 = 0$

$\Rightarrow \quad (2r - 1)(r - 2) = 0 \Rightarrow r = 2, \frac{1}{2}$

When $a = 10, r = 2, \frac{a}{r} = 5, a = 10, ar = 20$

When $a = 10, r = \frac{1}{2}, \frac{a}{r} = 20, a = 10, ar = 5$

Hence, the numbers are 5, 10, 20 or 20, 10, 5.

S62. Let a be the first term and d be the common ratio of A.P.

Let the numbers be $a - d$, a, $a + d$.

Now, $(a - d) + a + (a + d) = 21$

$\Rightarrow \quad 3a = 21 \Rightarrow a = 7$

Now, $a - d$, $a - 1$, $a + d + 1$ are in G.P.

$\Rightarrow$ $7 - d$, 6, $8 + d$ are in G.P. $[\because a = 7]$

$\Rightarrow \quad (6)^2 = (7 - d)(8 + d)$

$\Rightarrow \quad 36 = 56 - d - d^2$

$\Rightarrow \quad d^2 + d - 20 = 0$

$\Rightarrow \quad (d + 5)(d - 4) = 0 \Rightarrow d = 4, -5$

hence required numbers are 3,7,11 when a=7, d=4 and 12,7,2 when a=7, d=-5

Hence, the numbers are 3, 7, 11 or 12, 7, 2.

S63. Let a be the first term and r be the common ratio of G.P.

Here, $a = 2$, $r = 3$, $S_n = 728$

(i) $S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow 728 = \frac{2(3^n - 1)}{3 - 1}$

$\Rightarrow \quad 3^n = 729 = 3^6 \Rightarrow n = 6$

(ii) Here, $a = 1$, $r = 2$, $S_n = 255$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$255 = \frac{1 \cdot (2^n - 1)}{2 - 1} \Rightarrow 2^n = 256 = 2^8$$

$\Rightarrow \quad n = 8$

S64. First sequence : 2, 4, 8, 16, 32

Second sequence : 128, 32, 8, 2, $\frac{1}{2}$

New sequence after finding the products of the corresponding terms:

$$2 \times 128,\ 4 \times 32,\ 8 \times 8,\ 16 \times 2,\ 32 \times \frac{1}{2}$$

or 256, 128, 64, 32, 16

Now, $a = 256$

$$r = \frac{128}{256} = \frac{1}{2}$$

$$n = 5$$

Now, $$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_5 = \frac{256\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = \frac{256\left[1-\frac{1}{32}\right]}{\frac{1}{2}}$$

$$= 512 \times \frac{31}{32} = 496.$$

S65. Here, $S_n = 8 + 88 + 888 + 8888 + \ldots + n$ terms

$S_n = 8\,[1 + 11 + 111 + 1111 + \ldots + n$ terms]

$= \frac{8}{9}\,[9 + 99 + 999 + 9999 + \ldots + n$ terms]

$= \frac{8}{9}\,[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots + n$ terms]

$= \frac{8}{9}\,[(10 + 10^2 + 10^3 + \ldots + n$ terms$) - n]$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10}{9}(10^n - 1) - n\right].$$

S66. For the G.P. $T_4 = x,\quad T_{10} = y,\quad T_{16} = z$

Let the first term = a and common ratio = r

i.e., $T_4 = ar^3,\quad T_{10} = ar^9,\quad T_{16} = ar^{15}$

To Prove : x, y, z are in G.P.

or $y^2 = xz$

L.H.S. = $y^2 = (ar^9)^2 = a^2r^{18}$

R.H.S. = $xz = (ar^3)\,(ar^{15}) = a^2r^{18}$

L.H.S. = R.H.S.

Hence, x, y, z are in G.P.

S67. Let a be the first term and r the common ratio. Also

$$S_2 = -4$$
$$T_5 = 4T_3$$

$\Rightarrow$ $$\frac{a(1-r^2)}{1-r} = -4$$

or $$a(1 + r) = -4 \quad \ldots \text{(i)}$$
$$ar^4 = 4ar^2$$

or $$r^2 = 4$$

or $$r = \pm 2 \quad \ldots \text{(ii)}$$

From Eq. (i) and (ii), we get

When $$r = 2$$
$$a(1 + 2) = -4$$

or $$a = \frac{-4}{3}$$

Thus, the sequence is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$

From Eq. (i) and (ii), we get

When $$r = -2$$
$$a(1 - 2) = -4$$

or $$a = 4$$

Thus, the sequence is 4, −8, 16, −32, 64, ...

Hence, the required G.P. is $\left[\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\right]$ or [4, −8, 16, −32, 64, ...].

S68. Let first term = a and common ratio = r for the given G.P.

Here, $$a = 729$$
$$T_7 = 64$$

Let $$\text{Common ratio} = r$$
$$T_7 = 64$$

$\Rightarrow$ $$ar^6 = 64$$

or $$729r^6 = 64$$

or $$r^6 = \frac{64}{729}$$

or $$r^6 = \left(\frac{2}{3}\right)^6$$

or $$r = \frac{2}{3} < 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Hence $$S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$$

$$= 729 \times \frac{3}{1} \times \frac{(3^7 - 2^7)}{3^7}$$

$$= 3^7 \times (3^7 - 2^7)$$

$$= 2187 - 128 = 2059.$$

S69. Let first term = a and common ratio = r for the given G.P.

Given, $3, 3^2, 3^3, \ldots = 120$

Now, $a = 3$

$r = 3$

Let, Number of terms = n.

Also, $$\frac{a(r^n - 1)}{r-1} = 120$$

or $$\frac{3[3^n - 1]}{3-1} = 120$$

or $$3^n - 1 = \frac{120 \times 2}{3}$$

or $3^n - 1 = 80$

or $3^n = 81 = 3^4$

Hence, $n = 4$.

S70. Here, $t_{m+n} = p,\ T_{m-n} = q$

$\Rightarrow$ $AR^{m+n-1} = p$ and $AR^{m-n-1} = q$

On dividing the equations, we get

$$R^{2n} = \frac{p}{q} \quad \Rightarrow \quad R = \left[\frac{p}{q}\right]^{1/2n}$$

$$t_m = AR^{m-1} = AR^{m+n-1} \cdot R^{-n}$$

$$= p \cdot \left[\frac{p}{q}\right]^{-n/2n} = p \cdot \left[\frac{p}{q}\right]^{-1/2}$$

$$= p^{1/2} \cdot q^{1/2} = \sqrt{pq}\,.$$

S71. a, b, c are in A.P. $\Rightarrow$ $2b = a + c$ ... (i)

b, c, d are in G.P. $\Rightarrow$ $c^2 = bd$ ... (ii)

$\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.

$\Rightarrow$ $\frac{2}{d} = \frac{1}{c} + \frac{1}{e} = \frac{e+c}{ce}, d = \frac{2ce}{c+e}$ ... (iii)

Now, $c^2 = bd = \left(\frac{a+c}{2}\right) \cdot \left(\frac{2ce}{c+e}\right)$ [From (i) and (iii)]

$\Rightarrow$ $c = \frac{(a+c)e}{c+e}$

$\Rightarrow$ $c^2 + ce = ae + ce \Rightarrow c^2 = ae$

$\Rightarrow$ a, c, e are in G.P.

S72. We have: $t_n = 2^n + 3n.$

$\therefore$ $t_1 = 2^1 + 3.1;$

$t_2 = 2^2 + 3.2;$

$t_3 = 2^3 + 3.3;$

....

$t_n = 2^n + 3.\,n$

Adding column wise, we get:

$$S_n = (t_1 + t_2 + t_3 + \ldots\ldots + t_n)$$
$$= (2^1 + 2^2 + 2^3 + \ldots + 2^n) + 3\,(1 + 2 + 3 + \ldots + n)$$
$$= \frac{(2)(2^n - 1)}{(2-1)} + 3\left\{\frac{n(n+1)}{2}\right\} = \left\{\frac{4(2^n - 1) + 3n^2 + 3n}{2}\right\}.$$

S73. Given, .7 + .77 + .777 + ... to n terms

$$= 7 \times \{.1 + .11 + .111 + \ldots \text{ to } n \text{ terms}\}$$
$$= \frac{7}{9} \times \{.9 + .99 + .999 + \ldots \text{ to n terms}\}$$
$$= \frac{7}{9} \times \{(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots \text{ to } n \text{ terms}\}$$
$$= \frac{7}{9} \times [n - \{0.1 + 0.01 + 0.001 + \ldots \text{ to } n \text{ terms}\}]$$

$$= \frac{7}{9} \times \left[n - \left\{ \frac{(0.1)\{1-(0.1)^n\}}{(1-0.1)} \right\} \right] \quad \left[\because \; S_n = \frac{a(1-r^n)}{(1-r)}, \text{since } r<1 \right]$$

$$= \frac{7}{9} \times \left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right) \right] = \frac{7}{81}\left(9n - 1 + \frac{1}{10^n}\right).$$

S74. Given, 3 + 33 + 333 + ... to n terms

$$= 3 \times \{1 + 11 + 111 + \text{... to } n \text{ terms}\}$$

$$= \frac{3}{9} \times \{9 + 99 + 999 + \text{...to } n \text{ terms}\}$$

$$= \frac{3}{9} \times \{(10-1) + (10^2-1) + (10^3-1) + \text{... to } n \text{ terms}\}$$

$$= \frac{3}{9} \times [(10 + 10^2 + 10^3 + \text{... to } n \text{ terms}) - n]$$

$$= \frac{3}{9} \times \left[\left\{ \frac{(10)(10^n-1)}{(10-1)} \right\} - n \right] = \frac{1}{27}(10^{n+1} - 9n - 10).$$

S75. Let a be the first term and r be the common ratio of the G.P.

Then, $t_4 = \frac{1}{27} \Rightarrow ar^3 = \frac{1}{27}$ ... (i)

and $t_7 = \frac{1}{729} \Rightarrow ar^6 = \frac{1}{729}$ ... (ii)

Dividing Eq. (ii) by (i), we get:

$$\frac{ar^6}{ar^3} = \frac{27}{729} \Rightarrow r^3 = \left(\frac{1}{3}\right)^3 \Rightarrow r = \frac{1}{3}$$

Putting $r = \frac{1}{3}$ in (i)

We get, $a\left(\frac{1}{3}\right)^3 = \frac{1}{27} \Rightarrow a = 1.$

Now, sum of n terms of the G.P.

$$S_n = \frac{a(1-r^n)}{(1-r)} = \frac{1\left\{1-\left(\frac{1}{3}\right)^n\right\}}{\left(1-\frac{1}{3}\right)} = \frac{3}{2}\left(1 - \frac{1}{3^n}\right). \quad \left[\because \; r = \frac{1}{3} < 1\right]$$

S76. Let the G.P. be a, ar, ar^2, ...

then $a + ar + ar^2 + \ldots = 15$

and $a^2 + a^2r^2 + a^2r^4 + \ldots = 45$

$\Rightarrow \quad \dfrac{a}{1-r} = 15$ and $\dfrac{a^2}{1-r^2} = 45$

$\Rightarrow \quad a = 15(1 - r)$ ... (i)

$\Rightarrow \quad a^2 = 45(1 - r^2)$ ... (ii)

$\Rightarrow \quad [15(1 - r)]^2 = 45\ (1 - r^2)$

$\Rightarrow \quad 225(1 - r)^2 = 45(1 - r)(1 + r)$

$\Rightarrow \quad 5(1 - r) = (1 + r)$

$\Rightarrow \quad r = \dfrac{2}{3}$

From (i) $\quad a = 15\left(1 - \dfrac{2}{3}\right) = 5$

Hence the G.P. is $5, \dfrac{10}{3}, \dfrac{20}{9}, ...$

S77. Let a be the first term and r be the common ratio of G.P.

Let the G.P. be a, ar, ar^2,....

Now a + ar = 15

$\Rightarrow \quad a(1 + r) = 15$ ... (i)

and $a = ar + ar^2 + ar^3 ... = \dfrac{ar}{1-r}$

$\Rightarrow \quad 1 - r = r \quad \Rightarrow \quad r = \dfrac{1}{2}$

from (i) a $\left(1 + \dfrac{1}{2}\right) = 15 \quad \Rightarrow \quad a = 10$

Hence, the G.P. is $10, 5, \dfrac{5}{2}, ...$

S78. (i) Given, $\quad 9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} ... = 9^{1/3\ +\ 1/9\ +\ 1/27\ +\ ...}$

$= 9^{\frac{1/3}{1-1/3}} = 9^{1/2} = 3 \qquad \left[S_\infty = \dfrac{a}{1-r}\right]$

$\left[\text{The terms in exponent form an infinite G.P. with } a = \dfrac{1}{3}, r = \dfrac{1}{3}\right]$

(ii) Similarly, $\quad 6^{1/2} \cdot 6^{1/4} \cdot 6^{1/8} ... = 6^{1/2\ +\ 1/4\ +\ 1/8\ +\ ...} = 6^{\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)}}$

$$\left[\text{The terms in exponent form an infinite G.P. with } a = \frac{1}{2}, r = \frac{1}{2}\right]$$

$= 6^1 = 6.$

S79. Let $\frac{a}{r}$, a, ar be the first three terms of the G.P. Then,

$$\frac{a}{r} + ar + r = \frac{13}{12} \qquad \text{... (i)}$$

and $$\left(\frac{a}{r}\right)(a)(ar) = -1 \qquad \text{... (ii)}$$

From Eq. (ii), we get $a^3 = -1$, *i.e.*, $a = -1$ (considering only real roots)

Substituting $a = -1$ in Eq. (i), we have

$$-\frac{1}{r} - 1 - r = \frac{13}{12} \quad \text{or} \quad 12r^2 + 25r + 12 = 0$$

This is a quadratic in r, solving, we get

$$r = -\frac{3}{4} \quad \text{or} \quad -\frac{4}{3}$$

Thus, the three terms of G.P. are: $\frac{4}{3}, -1, \frac{3}{4}$ for $r = \frac{-3}{4}$ and $\frac{3}{4}, -1, \frac{4}{3}$ for $r = \frac{-4}{3}$.

S80. Here, $a = 1$ and $r = \frac{2}{3}$

Therefore, $$S_n = \frac{a(1-r^n)}{1-r} = \frac{\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}} = 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

In particular, $$S_5 = 3\left[1-\left(\frac{2}{3}\right)^5\right] = 3 \cdot \frac{211}{243} = \frac{211}{81}.$$

S81. Let n be the number of terms needed.

Given that, $a = 3$, $r = \frac{1}{2}$ and $S_n = \frac{3069}{512}$

Since, $$S_n = \frac{a(1-r^n)}{1-r}$$

Therefore, $$\frac{3069}{512} = \frac{3\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = 6\left(1-\frac{1}{2^n}\right)$$

or $$\frac{3069}{3072} = 1 - \frac{1}{2^n}$$

or $\frac{1}{2^n} = 1 - \frac{3069}{3072} = \frac{3}{3072} = \frac{1}{1024}$

or $2^n = 1024 = 2^{10}$

which gives $n = 10$.

Hence, $n = 10$.

S82. Let $S_n = .6 + .66 + .666 + \dots$ to n terms

$= 6\,[.1 + .11 + .111 + \dots \text{ to } n \text{ terms}]$

$$= \frac{6}{9}\left[\frac{9}{10} + \frac{99}{10^2} + \frac{999}{10^3} + \dots \text{ to } n \text{ terms}\right]$$

$$= \frac{6}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \dots \text{ to } n \text{ terms}\right]$$

$$= \frac{6}{9}\left[n \times 1 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots \text{ to } n \text{ terms}\right)\right]$$

$$= \frac{6}{9}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right]$$

$$= \frac{6}{9}\left[n - \frac{1}{10} \times \frac{10}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

$$= \frac{6}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

$$= \frac{6}{81}\left(9n - 1 + \frac{1}{10^n}\right).$$

S83. Let $S_n = 5 + 55 + 555 + \dots$ to n terms

$= 5\,[1 + 11 + 111 + \dots \text{ to } n \text{ terms}]$

$$= \frac{5}{9}\,[9 + 99 + 999 + \dots \text{ to } n \text{ terms}]$$

$$= \frac{5}{9}\,[(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ to } n \text{ terms}]$$

$$= \frac{5}{9}\,[(10 + 10^2 + 10^3 + \dots + n \text{ terms}) - n \times 1]$$

$$= \frac{5}{9}\left[\frac{10\,(10^n - 1)}{10 - 1} - n\right] = \frac{5}{9}\left[\frac{10^{n+1} - 10}{9} - n\right]$$

$$= \frac{5}{81}\,[10^{n+1} - 9n - 10].$$

S84. Given, $$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

or $$\frac{(a+bx)+(a-bx)}{(a+bx)-(a-bx)} = \frac{(b+cx)+(b-cx)}{(b+cx)-(b-cx)} = \frac{(c+dx)+(c-dx)}{(c+dx)-(c-dx)}$$

or $$\frac{2a}{2bx} = \frac{2b}{2cx} = \frac{2c}{2dx}$$

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Hence, a, b, c and d are in G.P.

S85. Let the even number of terms be

$$a, ar, ar^2, ar^3, ..., ar^{2n}$$

Now, $a + ar + ar^2 + ... + ar^{2n} = 5\,[a + ar^2 + ar^4 + ... + ar^{2n-1}]$

or $$\frac{a(r^{2n}-1)}{r-1} = 5a\left[\frac{(r^2)^n - 1}{r^2 - 1}\right]$$

or $$\frac{1}{r-1} = \frac{5}{r^2-1}$$

or $$r^2 - 1 = 5\,(r - 1)$$

or $$r + 1 = 5$$

Hence, $$r = 4.$$

S86. Let $$S_n = 315, \quad a = 5, \quad r = 2$$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{ar^n - a}{r-1}$$

$$= \frac{r\,a\,r^{n-1} - a}{r-1} = \frac{rl - a}{r-1}$$

Where l = Last term = T_n

or $$315 = \frac{2l-5}{2-1}$$

or $$315 = 2l - 5$$

or $$2l = 315 + 5 = 320$$

or $$l = \frac{320}{2} = 160$$

Now, $$l = T_n = a\,r^{n-1}$$

or $$160 = 5\,(2)^{n-1}$$

or $$2^{n-1} = \frac{160}{5} = 32$$

or $$2^{n-1} = (2)^5$$

or $$n - 1 = 5$$

i.e., $$n = 6$$

Hence, the last term = 160

and the number of terms = 6.

S87. Given, $$f(x + y) = f(x) \cdot f(y) \quad \text{for all} \quad x, y \in N$$

$$f(1) = 3$$

and $$\sum_{x=1}^{n} f(x) = 120$$

For $$x = 1, \quad y = 1$$

$$f(1 + 1) = f(1)\,f(1)$$

$$f(2) = 3 \times 3 = 9$$

For $$x = 1, \quad y = 2$$

$$f(1 + 2) = f(1)\,f(2)$$

$$= 3 \times 9 = 27$$

$$f(3) = 27$$

For $$x = 1, \quad y = 3$$

$$f(1 + 3) = f(1)\,f(3)$$

$$= 3 \times 27 = 81$$

$$f(4) = 81$$

Now, $$\text{L.H.S.} = \sum_{x=1}^{n} f(x)$$

$$= f(1) + f(2) + f(3) + \ldots + f(n)$$

$$= 3 + 9 + 27 + 81 + \ldots + n \text{ terms}$$

$$= \frac{3\,(3^n - 1)}{3 - 1} = \frac{3}{2}\,(3^n - 1)$$

Now, $$\sum_{x=1}^{n} f(x) = 120$$

$\therefore$ $$\frac{3}{2}\,(3^n - 2) = 120$$

$\Rightarrow \qquad 3^n - 1 = 80$

$\Rightarrow \qquad 3^n = 81 = 3^4$

Hence, $\qquad n = 4.$

S88. Let r be the common ratio of the G.P. a, b, c and d.

Then, $\qquad b = ar, \quad c = ar^2 \quad \text{and} \quad d = ar^3$

$$\text{L.H.S.} = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$$
$$= (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$$
$$= a^4r^2(1 + r^2 + r^4)^2$$

and
$$\text{R.H.S.} = (ab + bc + cd)^2$$
$$= (a^2r + a^2r^3 + a^2r^5)^2$$
$$= a^4r^2(1 + r^2 + r^4)^2.$$

Hence, $\qquad (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2.$

S89. Consider the G.P. a, ar, ar^2, ...

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Sum of terms from $(n + 1)^{\text{th}}$ to $(2n)^{\text{th}}$ terms

$$= S_{2n} - S_n$$
$$= \frac{a(r^{2n} - 1)}{r - 1} - \frac{a(r^n - 1)}{r - 1}$$

Now, $\qquad \text{Required ratio} = \dfrac{S_n}{S_{2n} - S_n}$

$$= \frac{\dfrac{a(r^n - 1)}{r - 1}}{\dfrac{a(r^{2n} - 1)}{r - 1} - \dfrac{a(r^n - 1)}{r - 1}}$$

$$= \frac{a(r^n - 1)}{a(r^{2n} - 1) - a(r^n - 1)}$$

$$= \frac{r^n - 1}{r^{2n} - 1 - r^n + 1}$$

$$= \frac{r^n - 1}{r^{2n} - r^n}$$

$$= \frac{r^n - 1}{r^n(r^n - 1)} = \frac{1}{r^n} = \text{R.H.S.}$$

S90. Let r be the common ratio of the given G.P. Then,

$b = n^{\text{th}}$ term $= ar^{n-1}$

$$\Rightarrow \quad r^{n-1} = \frac{b}{a} \quad \Rightarrow \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$$

Now, $\quad P$ = Product of the first n terms

$\Rightarrow \quad P = a \cdot ar \cdot ar^2 \ldots ar^{n-1}$

$\Rightarrow \quad P = a^n r^{1+2+3+\ldots+(n-1)}$

$$\Rightarrow \quad P = a^n r^{\frac{n(n-1)}{2}}$$

$$\Rightarrow \quad P = a^n \left\{\left(\frac{b}{a}\right)^{\frac{1}{n-1}}\right\}^{\frac{n(n-1)}{2}} = a^n \left(\frac{b}{a}\right)^{n/2}$$

$$= a^{n/2} b^{n/2} = (ab)^{n/2}$$

$\therefore \quad P^2 = [(ab)^{n/2}]^2 = (ab)^n$

$= (a \cdot b)^n$

Hence, $\quad P^2 = (ab)^n$.

S91. 1^{st} G.P.. : $\quad a, ar, ar^2, \ldots, ar^{n-1}$

2^{nd} G.P.. : $\quad A, AR, AR^2, \ldots, AR^{n-1}$

The sequence formed after multiplying the corresponding terms of the sequence is

$$(aA), (aA)(rR), (aA)\, r^2R^2, \ldots, (aA)\, r^{n-1}R^{n-1}$$

Here
$$\frac{T_2}{T_1} = \frac{(aA)rR}{aA} = rR$$

$$\frac{T_3}{T_2} = \frac{(aA)r^2R^2}{(aA)(rR)} = rR$$

$$\frac{T_4}{T_3} = \frac{(aA)(r^3R^3)}{(aA)(r^2R^2)} = rR$$

....

....

Since, the ratios of two succeeding terms are the same, the resulting sequence is also a G.P.

The common ratio of new G.P. = (rR).

S92. The equation $px^2 + 2qx + r = 0$ has roots gven by

$$x = \frac{-2q \pm \sqrt{4q^2 - 4rp}}{2p}$$

Since, p, q, r are in G.P. $q^2 = pr$. Thus, $x = \frac{-q}{p}$ but $\frac{-q}{p}$ is also root of $dx^2 + 2ex + f = 0$

$$d\left(\frac{q}{p}\right)^2 + 2e\left(\frac{q}{p}\right) + f = 0$$

or $$dq^2 - 2eqp + fp^2 = 0 \quad \text{... (i)}$$

Dividing Eq. (i) by pq^2 and using $q^2 = pr$, we get

$$\frac{d}{p} - \frac{2e}{q} + \frac{fp}{pr} = 0 \quad \text{or} \quad \frac{2e}{q} = \frac{d}{p} + \frac{f}{r}$$

Hence, $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

S93. Let a be the first term and d be the common difference of given A.P.

Here,
$$a_p = a + (p-1)d \quad \text{... (i)}$$
$$a_q = a + (q-1)d \quad \text{... (ii)}$$
$$a_r = a + (r-1)d \quad \text{... (iii)}$$
$$a_s = a + (s-1)d \quad \text{... (iv)}$$

Given, a_p, a_q, a_r and a_s are in G.P.

So,
$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q} = \frac{q-r}{p-q} \quad \text{... (v)}$$

Similarly,
$$\frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{a_r - a_s}{a_q - a_r} = \frac{r-s}{q-r} \quad \text{... (vi)}$$

Hence, by Eq. (v) and (vi)

$$\frac{q-r}{p-q} = \frac{r-s}{q-r},$$

i.e., $p - q$, $q - r$ and $r - s$ are in G.P.

S94. This is not a G.P., however, we can relate it to a G.P. by writing the terms as

$$S_n = 7 + 77 + 777 + 7777 + \dots \text{ to } n \text{ terms}$$

$$= \frac{7}{9}[9 + 99 + 999 + 9999 + \dots \text{ to } n \text{ terms}]$$

$$= \frac{7}{9}[(10-1) + (10^2-1) + (10^3-1) + (10^4-1) + \dots n \text{ terms}]$$

$$= \frac{7}{9}[(10 + 10^2 + 10^3 + \dots n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ terms})]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1} - n\right] = \frac{7}{9}\left[\frac{10(10^n-1)}{9} - n\right].$$

S95.

$$S_1 = \frac{1}{1-1/2} = 2,$$

$$S_2 = \frac{2}{1-1/3} = 3$$

$$S_3 = \frac{3}{1-1/4} = 4 \qquad \left[S_\infty = \frac{a}{1-r}\right]$$

$$\vdots$$

$$S_n = \frac{n}{1-1/n+1} = n+1$$

$\Rightarrow$ $S_1 + S_2 + \ldots + S_n = 2 + 3 + 4 + \ldots + (n + 1)$

$$= \frac{n}{2}[2(2) + (n-1)\cdot 1] \qquad [\because \text{ this is an A.P. with } a = 2,\ d = 1]$$

$$= \frac{n(n+3)}{2}.$$

S96. Let a be the first term and r be the common ratio of given G.P.

Here, $$S_1^2 + S_2^2 = \left[\frac{a(r^n-1)}{r-1}\right]^2 + \left[\frac{a(r^n-1)(r^n+1)}{r-1}\right]^2$$

$$= \left[\frac{a(r^n-1)}{r-1}\right]^2 [1+(r^n+1)^2] \qquad \ldots \text{(i)}$$

$$S_1(S_2 + S_3) = S_1\left[\frac{a(r^{2n}-1)}{r-1} + \frac{a(r^{3n}-1)}{r-1}\right]$$

$$= S_1\left[\frac{a(r^n-1)(r^n+1)}{r-1} + \frac{a(r^n-1)(r^{2n}+r^n+1)}{r-1}\right]$$

$$= \frac{a(r^n-1)}{r-1}\cdot\frac{(r^n+1)}{r-1}\ [r^n + 1 + r^{2n} + r^n + 1]$$

$$= \left[\frac{a(r^n-1)}{r-1}\right]^2 [1+(r^{2n}+2r^n+1)]$$

$$= \left[\frac{a(r^n-1)}{r-1}\right]^2 [1+(r^{2n}+2r^n+1)]$$

$$= \left[\frac{a(r^n-1)}{r-1}\right]^2 [1+(r^n+1)^2] \qquad \ldots \text{(ii)}$$

From (i) and (ii) we get $S_1^2 + S_2^2 = S_1(S_2 + S_3)$

(ii)

$$S_1[S_3 - S_2] = S_1\left[\frac{a(r^{3n}-1)}{r-1} - \frac{a(r^{2n}-1)}{r-1}\right]$$

$$= \left[\frac{a(r^n-1)(r^n+1)}{r-1} + \frac{a(r^n-1)(r^{2n}-r^n-1)}{r-1}\right]$$

$$= \left[\frac{a(r^n-1)}{r-1}\right]^2 [r^{2n} + r^n + 1 - (r^n + 1)]$$

$$= \left[\frac{a(r^{2n}-r^n)}{r-1}\right]^2 \quad \text{... (i)}$$

$$S_2 - S_1 = \frac{a(r^{2n}-1)}{r-1} - \frac{a(r^n-1)}{r-1}$$

$$(S_2 - S_1)^2 = \left[\frac{a(r^{2n}-r^n)}{r-1}\right]^2 \quad \text{... (ii)}$$

From (i) and (ii) $S_1[S_3 - S_2] = (S_2 - S_1)^2$.

S97. Let A be the first term and R the common ratio of the G.P. then,

$$AR^{p-1} = a \quad \text{... (i)}$$

$$AR^{q-1} = b \quad \text{... (ii)}$$

$$AR^{r-1} = c \quad \text{... (iii)}$$

On dividing (i) by (ii) and (ii) by (iii), we get

$$\frac{a}{b} = R^{p-q} \quad \text{and} \quad \frac{b}{c} = R^{q-r}$$

$$\Rightarrow \quad R = \left(\frac{a}{b}\right)^{1/p-q} = \left(\frac{b}{c}\right)^{1/q-r} \quad \text{... (iv)}$$

Let B be the first term and D the common difference of the A.P.

$$\Rightarrow \quad B + (p-1)D = a \quad \text{... (v)}$$

$$B + (q-1)D = b \quad \text{... (vi)}$$

$$B + (r-1)D = c \quad \text{... (vii)}$$

On subtracting (vi) from (v) and (vii) from (vi) we get

$$a - b = (p-q)D,\ b - c = (q-r)D$$

$$\Rightarrow \quad \frac{1}{p-q} = \frac{D}{a-b}, \frac{1}{q-r} = \frac{D}{b-c}$$

from (iv) $\left(\frac{a}{b}\right)^{1/p-q} = \left(\frac{b}{c}\right)^{1/q-r}$

$\Rightarrow$ $\left(\frac{a}{b}\right)^{\frac{D}{a-b}} = \left(\frac{b}{c}\right)^{\frac{D}{b-c}}$

$\Rightarrow$ $\left(\frac{a}{b}\right)^{\frac{1}{a-b}} = \left(\frac{b}{c}\right)^{\frac{1}{b-c}}$

$\Rightarrow$ $\left(\frac{a}{b}\right)^{b-c} = \left(\frac{b}{c}\right)^{a-b}$

$\Rightarrow$ $\frac{a^{b-c}}{b^{b-c}} = \frac{b^{a-b}}{c^{a-b}}$

$\Rightarrow$ $a^{b-c} \cdot c^{a-b} = b^{b-c+a-b}$

$a^{b-c} \cdot c^{a-b} = b^{-(c-a)}$

$\Rightarrow$ $a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1.$

S98. Let the number be $a - d$, a, a + d.

Now, $(a - d) + a + (a + d) = 18$ $\Rightarrow$ $3a = 18$ $\Rightarrow$ $a = 6$

Now, $a - d + 4$, $a + 4$, $a + d + 36$ are in G.P.

$\Rightarrow$ $10 - d$, 10, $42 + d$ are in G.P. [$\because a = 6$]

$\Rightarrow$ $(10)^2 = (10 - d)(42 + d)$

$\Rightarrow$ $100 = 420 - 32d - d^2$

$\Rightarrow$ $d^2 + 32d - 320 = 0$

$\Rightarrow$ $(d + 40)(d - 8) = 0$

$\Rightarrow$ $d = 8, -40$

when $a = 6$, $d = 8$; $a - d = -2$,

$a = 6$, $a + d = 14$

when $a = 6$, $d = -40$; $a - d = 46$,

$a = 6$, $a + d = -34$

Hence, the numbers are 46, 6, – 34. or -2,6,14

S99. Let a be the first term and r be the common ratio of G.P.

Then, $b = ar$, $c = ar^2$, $d = ar^3$

(a) $(b^n + c^n)^2 = (a^n r^n + a^n r^{2n})^2$

$= a^{2n} r^{2n} (1 + r^n)^2$ ... (i)

$(a^n + b^n)(c^n + d^n) = (a^n + b^n r^n)(a^n r^{2n} + a^n r^{3n})$

$= a^n (1 + r^n) a^n r^{2n} (1 + r^n)$

$$= a^{2n}r^{2n}(1 + r^n)^2 \quad \text{... (ii)}$$

From (i) and (ii)

$$(b^n + c^n)^2 = (a^n + b^n)(c^n + d^n)$$

$\Rightarrow$ $a^n + b^n$, $b^n + c^n$, $c^n + d^n$ are in G.P.

(b)
$$(ab + bc + cd)^2 = (a \cdot ar + ar \cdot ar^2 + ar^2 \cdot ar^3)^2$$
$$= (a^2r + a^2r^3 + a^2r^5)^2$$
$$= a^4r^2(1 + r^2 + r^4)^2 \quad \text{... (i)}$$

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$$
$$= a^2(1 + r^2 + r^4)a^2r^2(1 + r^2 + r^4)$$
$$= a^4r^2(1 + r^2 + r^4)^2 \quad \text{... (ii)}$$

From (i) and (ii)

$$(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$$

$\Rightarrow$ $a^2 + b^2 + c^2$, $ab + bc + cd$, $b^2 + c^2 + d^2$ are in G.P.

S100. Let a be the first term and r be the common ratio of G.P.

Then, $b = ar$, $c = ar^2$, $d = ar^3$

(a)
$$a(b^2 + c^2) = a(a^2r^2 + a^2r^4)$$
$$= a^3r^2(1 + r^2) \quad \text{... (i)}$$
$$c(a^2 + b^2) = ar^2(a^2 + a^2r^2)$$
$$= a^3r^2(1 + r^2) \quad \text{... (ii)}$$

From (i) and (ii)

$$a(b^2 + c^2) = c(a^2 + b^2)$$

(b)
$$\frac{1}{a^2 - b^2} + \frac{1}{b^2} = \frac{1}{a^2 - a^2r^2} + \frac{1}{a^2r^2}$$

$$= \frac{1}{a^2(1 - r^2)} + \frac{1}{a^2r^2} = \frac{r^2 + 1 - r^2}{a^2r^2(1 - r^2)}$$

$$= \frac{1}{a^2r^2(1 - r^2)} \quad \text{... (i)}$$

$$\frac{1}{b^2 - c^2} = \frac{1}{a^2r^2 - a^2r^4} = \frac{1}{a^2r^2(1 - r^2)} \quad \text{... (ii)}$$

From (i) and (ii)

$$\frac{1}{a^2 - b^2} + \frac{1}{b^2} = \frac{1}{b^2 - c^2}.$$

S101. Let the G.P. of n terms be a, ar, ar^2, ..., where $r < 1$ (say).

Then $$S = \frac{a(1-r^n)}{1-r} \quad \text{... (i)}$$

Also $$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots \text{ to } n \text{ terms}$$

$$= \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n - 1\right]}{\frac{1}{r} - 1} \quad \left[\frac{1}{r} > 1 \text{ as } r < 1\right]$$

$$= \frac{1}{a} \cdot \frac{1-r^n}{r^n} \cdot \frac{r}{1-r}$$

$$= \frac{1-r^n}{ar^{n-1}(1-r)} \quad \text{... (ii)}$$

and $$P = (a)(ar)(ar^2) \ldots (ar^{n-1})$$

$$= a^n \cdot r^{1+2+3+\ldots+(n-1)}$$

$$= a^n \cdot r^{\frac{n-1}{2}(1+n-1)}$$

$$= a^n (r)^{\frac{(n-1)n}{2}} \quad \text{... (iii)}$$

$$\text{L.H.S.} = P^2 R^n$$

$$= a^{2n} r^{n(n-1)} \cdot \frac{(1-r^n)^n}{(ar^{n-1})(1-r)^n}$$

$$= \frac{a^n(1-r^n)^n}{(1-r)^n} = \left[\frac{a(1-r^n)}{1-r}\right]^n$$

$= S^n$. **Hence proved**. [Using Eq. (i)]

S102. Let a be the first term and r be the common ratio.

We have, $$S_3 = 16$$

$$\Rightarrow \quad \frac{a(1-r^3)}{1-r} = 16 \quad \text{... (i)}$$

$$\Rightarrow \quad S_6 - S_3 = 128$$

$$\Rightarrow \quad \frac{a(1-r^6)}{1-r} - 16 = 128$$

i.e., $$\frac{a(1-r^6)}{1-r} = 144 \quad \text{... (ii)}$$

$\frac{(ii)}{(i)}$ gives $\frac{1-r^6}{1-r^3} = \frac{144}{16} = \frac{9}{1}$

or $\frac{(1-r^3)(1+r^3)}{1-r^3} = \frac{9}{1}$

or $1 + r^3 = 9$

or $r^3 = 8$

or $r^3 = 2^3$

or $r = 2$

Thus, Common ratio = 2

As $r = 2$

$$S_3 = \frac{a(r^3-1)}{r-1} = 16$$

or $\frac{a(2^3-1)}{2-1} = 16$

or $\frac{a(8-1)}{1} = 16$

or $a(7) = 16$

or $a = \frac{16}{7}$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$= \frac{16}{7}\frac{(2^n-1)}{2-1}$$

$$= \frac{16}{7}(2^n-1).$$

S103. Let three terms in G.P. are

$$\frac{a}{r}, a, ar$$

Their product = $\frac{a}{r}, a, ar = 1$

or $a^3 = 1$

or $a = 1$

Again, $\frac{a}{r} + a + ar = \frac{39}{10}$

or $\frac{1}{r} + 1 + r = \frac{39}{10}$

or $$1 + r + r^2 = \frac{39}{10} r$$

or $$10 + 10r + 10r^2 = 39 r$$

or $$10r^2 - 29r + 10 = 0$$

or $$10r^2 - 25r - 4r + 10 = 0$$

or $$5r(2r - 5) - 2(2r - 5) = 0$$

or $$(2r - 5)(5r - 2) = 0$$

or $$r = \frac{2}{5}, \frac{5}{2}$$

Case I: When $r = \frac{2}{5}$, $a = 1$

Numbers are $\frac{1}{2/5}$, 1, $1 \times \frac{2}{5}$

or $$\frac{5}{2}, 1, \frac{2}{5}$$

Case II: When $r = \frac{5}{2}$, $a = 1$

Numbers are $\frac{a}{r}$, a, ar

or $$\frac{1}{5/2}, 1, 1 \times \frac{5}{2}$$

or $$\frac{2}{5}, 1, \frac{5}{2}$$

Hence, three numbers are $\frac{2}{5}, 1, \frac{5}{2}$.

or $$\frac{5}{2}, 1, \frac{2}{5}.$$

S104. Let the numbers be a, ar, ar^2

$$a + ar + ar^2 = 7$$

and $$a^2 + a^2r^2 + a^2r^4 = 21 \quad \text{... (i)}$$

$\Rightarrow$ $$a(1 + r + r^2) = 7 \quad \text{... (ii)}$$

and $$a^2(1 + r^2 + r^4) = 21$$

Squaring (i) and dividing by (ii),

$$\frac{a^2(1 + r + r^2)^2}{a^2(1 + r^2 + r^4)} = \frac{49}{21}$$

$\Rightarrow \qquad \dfrac{(1+r+r^2)^2}{1+r^2+r^4} = \dfrac{7}{3}$

Now, $\qquad 1 + r^2 + r^4 = (1 + r^2)^2 - r^2$

$= (1 + r^2 + r)(1 + r^2 - r) \qquad \ldots \text{(iii)}$

From (iii),

$$\frac{(1+r+r^2)^2}{(1+r+r^2)(1-r+r^2)} = \frac{7}{3}$$

$\Rightarrow \qquad 7(1 - r + r^2) = 3(1 + r + r^2)$

$\Rightarrow \qquad 4r^2 - 10r + 4 = 0$

$\Rightarrow \qquad 2r^2 - 5r + 2 = 0$

$\Rightarrow \qquad (r - 2)(2r - 1) = 0$

$\Rightarrow \qquad r = 2, \dfrac{1}{2}$

From (i), $\qquad a = \dfrac{7}{1+r+r^2}$

For $\qquad r = 2, \quad a = \dfrac{7}{7} = 1$

For $\qquad r = \dfrac{1}{2}, \quad a = \dfrac{7}{1+\frac{1}{2}+\frac{1}{4}} = 4$

When $a = 1$, $r = 2$, $a = 1$, $ar = 2$, $ar^2 = 4$

When $a = 4$, $r = \dfrac{1}{2}$, $a = 4$, $ar = 2$, $ar^2 = 1$

Hence, the terms are 1, 2, 4 or 4, 2, 1.

Date: 12/10/2021

Q1. Find the geometric mean of 7, 7^2, 7^3,...., 7^n.

Q2. Find the geometric mean between – 4 and –25.

Q3. Insert three geometric means between 1 and 256.

Q4. If G.M. between two numbers a and b is G and the two A.Ms. between them are p and q, then, prove that: $G^2 = (2p - q)(2q - p)$.

Q5. Find two number whose arithmetic mean is 34 and the geometric mean is 16.

Q6. If G be the geometric mean between two given numbers and A_1, A_2 be the two arithmetic means between them, prove that $G^2 = (2A_1 - A_2)(2A_2 - A_1)$.

Q7. If a, b, c are in A.P., x is the G.M. between a and b, y is the G.M. between b and c, then show that b^2 is the A.M. between x^2 and y^2.

Q8. If the A.M. of two numbers be A and G.M. be G, then find the numbers in terms of A and G.

Q9. Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b.

Q10. The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio $3 + 2\sqrt{2} : 3 - 2\sqrt{2}$.

Q11. Prove that the product of n G.Ms. between any two positive numbers is equal to n th power of the G.M. between them.

Q12. If x is the A.M. between a and b ; y is the A.M. between b and c while a, b, c are in G.P., prove that:

(a) $\frac{a}{x} + \frac{c}{y} = 2$ (b) $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$

Q13. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then prove that $\sqrt{\frac{y^3 + z^3}{xyz}} = 2$.

Q14. Prove that the G.M. of any two positive numbers a and b, $a < b$, is nearest to a.

Q15. Let a be a positive number such that the arithmetic mean of a and 2 exceeds their geometric mean by 1. Then, find the value of a.

Q16. If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

Q17. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Q18. The ratio of the A.M. and G.M. of two positive number a and b is $m : n$. show that:

$$a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2}).$$

Q19. If a is the A.M. of b and c and the two geometric means G_1 and G_2 are inserted between b and c, then prove that $G_1^3 + G_2^3 = 2abc$.

Date: 12/10/2021

S1. The geometric mean of the numbers 7, 7^2, 7^3, ..., 7^n is

$$G = (7, 7^2, 7^3, ..., 7^n)^{1/n} \qquad \left[\sum n = \frac{n(n+1)}{2}\right]$$

$$= \{7^{(1+2+3+...n)}\}^{1/n} = \left\{7^{\left(\frac{n(n+1)}{2}\right)}\right\}^{1/n} = 7^{\left(\frac{n(n+1)}{2n}\right)} = 7^{\frac{n+1}{2}}$$

S2. If b is GM of a and b, then $b = -\sqrt{ab}$. $(a, b < 0)$.

The G.M. between (–4) and (– 25)

$$= (-4)^{1/2}(-25)^{1/2} = (2i)(5i) = 10\,i^2 = -10.$$

S3. Let G_1, G_2, G_3 be the three G.Ms. between 1 and 256. Then, 1, G_1, G_2, G_3, 256 are in G.P.

Let r be the common ratio and 256 is the fifth term. Then,

$$256 = T_5 = 1(r)^{5-1} \Rightarrow r^2 = 16 \Rightarrow r = \pm 4$$

If $r = 4$, then $G_1 = (1)(4) = 4$, $G_2 = (1)(4)^2 = 16$, $G_3 = (1)(4)^3 = 64$

If $r = -4$, then $G_1 = (1)(-4) = -4$, $G_2 = (1)(-4)^2 = 16$, $G_3 = (1)(-4)^3 = -64$

$\therefore$ 4, 16, 64 or $-4, -16, -64$ are the three geometric means.

S4. We have $G^2 = ab$ ... (i) ($\because$ G is G.M. between a and b)

Also, p and q are two arithmetic means between a and b.

$\therefore$ a, p, q, b are in A.P.

$\Rightarrow$ $2p = a + q$ and $2q = p + b$

$\Rightarrow$ $2p - q = a$ and $2q - p = b$

Putting, $a = 2p - q$ and $b = 2q - p$ in (i), we get

$$G^2 = (2p - q)(2q - p)$$

S5. Let the two numbers be a and b.

Then, $\frac{a+b}{2} = 34$ and $\sqrt{ab} = 16$ $\Rightarrow$ $a + b = 68$ and $ab = 256$

$\therefore$ $(a - b)^2 = (a + b)^2 - 4ab = (68)^2 - 4(256) = 3600$

$\Rightarrow$ $a - b = 60$.

On solving $a + b = 68$ and $a - b = 60$, we get:

$$a = 64 \quad \text{and} \quad b = 4$$

Thus, the required numbers are 64 and 4.

S6. Let the two given numbers be a and b.

Then, $G^2 = ab$ ...(i)

Also, a, A_1, A_2, b are in A.P. $\Rightarrow$ $2A_1 = a + A_2$ and $2A_2 = A_1 + b$

$\Rightarrow$ $(2A_1 - A_2) = a$ and $(2A_2 - A_1) = b$

$\therefore$ $G^2 = ab = (2A_1 - A_2)(2A_2 - A_1)$.

$$G^2 = (2A_1 - A_2)(2A_2 - A_1)$$

S7. Since a, b, c are A.P. $\Rightarrow$ $2b = a + c$ ... (i)

x is the G.M. between a and b $\Rightarrow$ $x = \sqrt{ab}$ ... (ii)

y is the G.M. between b and c $\Rightarrow$ $y = \sqrt{bc}$ ... (iii)

Now, $x^2 + y^2 = ab + bc$ [Using (ii) and (iii)]

$= b(a + c) = b(2b) = 2b^2$ [Using (i)]

$\therefore$ b^2 is the A.M. between x^2 and y^2.

S8. Let a and b be the two numbers.

Then, quadratic equation with a and b as its roots is

$$x^2 - (a + b) + ab = 0$$

i.e. $x^2 - 2Ax + G^2 = 0$ $\left[\text{Using } A = \dfrac{a+b}{2}, G = \sqrt{ab}\right]$

The roots of this equation are:

$$x = \frac{2A \pm \sqrt{4A^2 - 4G^2}}{2} = A \pm \sqrt{(A+G)(A-G)}$$

Hence, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

S9. We know that G.M. between a and $b = \sqrt{ab}$ $[\because \ a, b > 0]$

$\Rightarrow$ $$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

$\Rightarrow$ $$a^{n+1} + b^{n+1} = (a^n + b^n)(ab)^{\frac{1}{2}}$$

$\Rightarrow$ $$a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} \cdot b^{n+\frac{1}{2}}$$

$$\Rightarrow \quad a^{n+1} - a^{n+\frac{1}{2}} \cdot b^{\frac{1}{2}} = a^{\frac{1}{2}} \cdot b^{n+\frac{1}{2}} - b^{n+1}$$

$$\Rightarrow \quad a^{n+\frac{1}{2}}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right) = b^{n+\frac{1}{2}}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)$$

$$\Rightarrow \quad a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}} \left[\because\ a \neq b \ \Rightarrow \ a^{\frac{1}{2}} - b^{\frac{1}{2}} \neq 0\right]$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = 1 = \left(\frac{a}{b}\right)^{0}$$

$$\Rightarrow \quad n + \frac{1}{2} = 0 \Rightarrow n = -\frac{1}{2}.$$

S10. Let two numbers are a and b. Then, G.M. between a and b = $\sqrt{ab}$

Given that Sum of two numbers a and b = 6 × (G.M. between a and b)

$$\Rightarrow \quad a + b = 6(\sqrt{ab}) \quad \Rightarrow \quad \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Applying componendo and dividendo, we get

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1} \quad \Rightarrow \quad \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{4}{2} \quad \Rightarrow \quad \frac{(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{2}}{1}$$

Again, applying componendo and dividendo, we get

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}+\sqrt{a}+\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\Rightarrow \quad \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \quad \Rightarrow \quad \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

Squaring both sides, we get $\frac{a}{b} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 \quad \Rightarrow \quad \frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}} \quad \Rightarrow \quad \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

Hence, the problem.

S11. Let $a, G_1, G_2, G_3, \ldots, G_n, b$ are in G.P. $[a, b > 0, (a \neq b)]$

Let r be the common ratio of this G.P. and b is $(n + 2)^{nd}$ term.

Now, $\quad b = T_{n+2} = ar^{n+1} \quad \Rightarrow \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Product of n G.Ms. between a and b = $G_1.G_2 \ldots G_n = ar \, . \, ar^2 \ldots ar^n$

$$= a^n r^{1+2+\ldots+n} = a^n \cdot r^{\frac{n}{2}[2(1)+(n-1)1]} = a^n r^{\frac{n(n+1)}{2}}$$

$$= a^n\left[\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right]^{\frac{n(n+1)}{2}} = a^n\left(\frac{b}{a}\right)^{\frac{n}{2}}$$

$$= a^{n/2}\cdot b^{n/2} = (\sqrt{ab})^n$$

$\Rightarrow$ Product of n G.Ms. between a and b = (G.M. between a and b)^n.

S12. x is the A.M. between a and b $\Rightarrow$ $x = \frac{a+b}{2}$ ... (i)

y is the A.M. between b and c $\Rightarrow$ $y = \frac{b+c}{2}$ ... (ii)

a, b, c are in G.P. $\Rightarrow$ $b^2 = ac$ ... (iii)

(a) From (iii): $b^2 = ac$ $\Rightarrow$ $b.b = ac \Rightarrow (2x - a)(2y - c) = ac$

[$\because$ From (i): $b = 2x - a$; From (ii): $b = 2x - c$]

$\Rightarrow$ $4xy - 4xc - 2ay = 0$ $\Rightarrow$ $xc + ay = 2xy \Rightarrow \frac{a}{x} + \frac{c}{y} = 2$

(b) From (i) and (ii) we get:

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c} = \frac{2(b+c)+2(a+b)}{ab+ac+bc+b^2}$$

$$= \frac{2(a+c+2b)}{ab+2b^2+bc}$$ [Using (iii): $b^2 = ac$]

$$= \frac{2(a+c+2b)}{b(a+c+2b)} = \frac{2}{b}.$$

$\therefore$ $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}.$

S13. Let a be the first term and r be the common ratio of given G.P.

Let the two required numbers be a and b.

Then, x is the A.M. between a and b.

$\Rightarrow$ a, y, z, b are in G.P. Let r be the common ratio of this G.P. Then, $y = ar$, $z = ar^2$

Also, $b = ar^3$ $\Rightarrow$ $r = \left(\frac{b}{a}\right)^{1/3}$

Now, $$\sqrt{\frac{y^3+z^3}{xyz}} = \frac{(ar)^3+(ar^2)^3}{\left(\frac{a+b}{2}\right)(ar)(ar^2)} = \frac{a^3r^3(1+r^3)}{a^2\left(\frac{a+b}{2}\right)r^3}$$

$$= \frac{2a\left(1+\frac{b}{a}\right)}{(a+b)} = 2. \qquad \left[\because\ r = \left(\frac{b}{a}\right)^{1/3} \Rightarrow r^3 = \frac{b}{a}\right]$$

S14. The G.M. of a and b is $\sqrt{ab}$.

We have: $\sqrt{ab} - a = \sqrt{a}(\sqrt{b} - \sqrt{a})$

and $b - \sqrt{ab} = \sqrt{b}(\sqrt{b} - \sqrt{a})$

Now a and b are positive and $a < b$ (given)

$\therefore \quad \sqrt{b} > \sqrt{a}$.

And so, $(\sqrt{b} > \sqrt{a})$ is positive.

$\therefore \quad \sqrt{b}\,(\sqrt{b} - \sqrt{a}) > \sqrt{a}(\sqrt{b} - \sqrt{a})$

$\Rightarrow \quad b - \sqrt{ab} > \sqrt{ab} - a$

i.e., the difference between G.M. and b is greater than the difference between G.M. and a. Hence, the G.M. of a and b is nearest to a.

S15. Let A be the A.M. and G be the G.M. between a and 2.

Then, $A = \frac{a+2}{2}$

and $G = \sqrt{2a}$

Now, $A - G = 1$

$\Rightarrow \quad \frac{a+2}{2} - \sqrt{2a} = 1 \quad \Rightarrow \quad \frac{a+2}{2} - 1 = \sqrt{2a}$

$\Rightarrow \quad \frac{a}{2} = \sqrt{2a} \quad \Rightarrow \quad \frac{a^2}{4} = 2a$

$\Rightarrow \quad a^2 - 8a = 0 \quad \Rightarrow \quad a(a-8) \quad \Rightarrow \quad a = 8. \qquad [\because \quad a \neq 0]$

S16. Given that A.M. $= \frac{a+b}{2} = 10$ ... (i)

and G.M. $= \sqrt{ab} = 8$ ... (ii)

From Eq. (i) and (ii), we

$a + b = 20$ ... (iii)

$ab = 64$ `... (iv)

Putting the value of a and b from Eq. (iii), (iv) in the identity $(a - b)^2 = (a + b)^2 - 4ab$, we get

$(a - b)^2 = 400 - 256 = 144$

or $a - b = \pm 12$ ... (v)

Solving Eq. (iii) and (v), we obtain

$$a = 4, \quad b = 16 \quad \text{or} \quad a = 16, \quad b = 4$$

Thus, the numbers a and b are 4, 16 or 16, 4 respectively.

S17. Let the two numbers between 3 and 81 be G_1 and G_2.

Thus, 3, G_1, G_2, 81 are in G.P.

Here, $a = 3$

$T_4 = 81$

Let r be the common ratio

Therefore, $ar^3 = 81$

or $3r^3 = 81$

or $r^3 = 27$

or $r = 3$

or $G_1 = 3 \times 3 = 9$

or $G_2 = 9 \times 3 = 27$

Hence, the required numbers are 9 and 27.

S18. Here,

$$\frac{A.M.}{G.M.} = \frac{m}{n}$$

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \quad \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n} \qquad \text{[Componendo and dividendo]}$$

or

$$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$$

$$\frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m-n}-\sqrt{m-n}} \qquad \text{[Componendo and dividendo]}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\frac{a}{b} = \frac{(m+n)+(m-n)+2\sqrt{(m+n)(m-n)}}{(m+n)+(m-n)-2\sqrt{(m+n)+(m-n)}} \quad \text{[Squaring both sides]}$$

$$= \frac{2m+2\sqrt{m^2-n^2}}{2m-2\sqrt{m^2-n^2}} \quad \Rightarrow \quad \frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

Hence, $a : b = (m+\sqrt{m^2-n^2}):(m-\sqrt{m^2-n^2})$.

S19. Since, a is the A.M. of b and c $\Rightarrow 2a = b + c$. ... (i)

$\therefore$ b, G_1, G_2, c are G.P. Let r be the common ratio of this G.P.

Then, $c = br^3 \Rightarrow r = \left(\frac{c}{b}\right)^{1/3}$

$\therefore$ $G_1 = br = b\left(\frac{c}{b}\right)^{1/3} = b^{2/3}c^{1/3} \Rightarrow G_1^{\,3} = (b^{2/3}\, c^{1/3})^3 = b^2c$

and $G_2 = br^2 = b\left\{\left(\frac{c}{b}\right)^{1/3}\right\}^2 = b^{1/3}c^{2/3} \Rightarrow G_2^{\,3} = (b^{1/3}\, c^{2/3})^3 = bc^2$

Now, $G_1^{\,3} + G_2^{\,3} = b^2c + bc^2 = bc\,(a + c) = bc\,(2a)$ [Using (i)]

$\Rightarrow$ $G_1^{\,3} + G_2^{\,3} = 2\,abc$.

Date: 12/10/2021

Q1. A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs. 5 every month, what amount will he pay in the 30^{th} installment.

Q2. The income of a person is Rs. 3,00,000 in the first year and he receives an increase of Rs. 10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

Q3. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by 1/2 km each succeeding hour. After how many hours will the second car overtake the first car, if both cars go non-stop.

Q4. The gate receipts at the show of "Baghbaan" amounted to Rs. 9,500 on the first night and showed a drop of Rs. 250 every succeeding night. If the operational expenses of the show are Rs. 2,000 a day, find on which night the show ceases to be profitable?

Q5. A person buys National Savings Certificates every year of value exceeding the last years purchase by Rs. 100. After 5 year he find that the total value of the certificate is Rs. 5250. Find the value of the certificates purchased by him:

(i) in the 1^{st} year (ii) in the 9^{th} year.

Q6. The ages of the students of a class form an A.P. whose common difference is 4 months. If the youngest student is 8 years old and the sum of the ages of all students of the class is 168 years, find the number of student in the class.

Q7. A man deposited $ 10000 in bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

Q8. The gate receipt at the show of "Rain-Coat" amounted to Rs. 6,500 on the first night and showed a drop of Rs. 110 every succeeding night. If the operational expenses of the show are Rs. 1000 a day. Find on which night the show ceases to be profitable.

Q9. A man buys a car for Rs. 3,00,000.He pays Rs. 1,50,000 in cash and agrees to pay the balance in yearly installments of Rs. 10,000 plus 10% interest on the unpaid amount, find the total amount he paid for the car.

Q10. A manufacturer of radio – set produced 600 units in the third year and 700 units in the seventh year. Assuming that the production uniformly increases by a fixed number every year, find

(a) the production in the first year (b) the total production in 7 years.

Q11. The sides of a right-angled triangle are in A.P. show that the sides are in the ratio 3 : 4 : 5.

Q12. On the first day strike of physicians in a hospital, the attendance of the O.P.D. was 750 patients. As the strike continued the attendance declined by 50 patients every day. Find from which day of the strike the O.P.D. would have no patient.

Q13. 150 workers were engaged to finish a job in a certain number of days, 4 workers dropped out on the second day, 4 more workers dropped out on the third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.

Q14. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Q15. A farmer buys a used scooter for $12,000. He pays $6000 cash and agrees to pay the balance in annual installements of $500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

Q16. Ram buys a scooter for $22000. He pays 4000 cash and agrees to pay the balance in annual installment of $1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Date: 12/10/2021

S1. Thus he pays Rs. 100, Rs. 105, Rs. 110, ...

Now, $a = 100, \quad d = 5$

To find T_{30}

We know that $T_n = a + (n - 1)\, d$

$$T_{30} = 100 + (30 - 1) \times 5$$
$$= 100 + 145 = 245$$

Hence, he will pay Rs. 245 in the 30th installment.

S2. Here, we have an A.P. with $a = 3{,}00{,}000$, $d = 10{,}000$ and $n = 20$.

Using the sum formula, we get,

$$S_{20} = \frac{20}{2}\,[6{,}00{,}000 + 19 \times 10{,}000]$$
$$= 10\,(7{,}90{,}000) = 79{,}00{,}000.$$

Hence, the person received Rs. 79,00,000 as the total amount at the end of 20 years.

S3. Let the second car overtake the first care after n hours. Then, the two cars travel the same distance in n hours.

Distance travelled by the first car in n hours = $10\,n$ km

Distance travelled by the second car in n hours = sum of n terms of an A.P. with first term 8 and common difference $\frac{1}{2}$.

$$= \frac{n}{2}\left[2 \times 8 + (n - 1) \times \frac{1}{2}\right] = \frac{n(n + 31)}{4}$$

When the second car overtakes the first car, we have

$$10n = \frac{n(n + 31)}{4}$$

$\Rightarrow$ $40n = n^2 + 31n \Rightarrow n(n - 9) = 0 \Rightarrow n = 9$ $[\because n \neq 0]$

Thus, the second car will overtake the first car in 9 hours.

S4. We have cost of gate receipt on the first night (a) = 9500;

Common difference (d) = – 250

The show will cease to be profitable on the night, when

$$2000 = 9500 + (n - 1)(-250)$$

$\Rightarrow$ $$2000 - 9500 = -250n + 250$$

$\Rightarrow$ $$2000 - 9500 - 250 = -250n$$

$\Rightarrow$ $$-7750 = -250n$$

$\Rightarrow$ $$n = \frac{7750}{250} = 31.$$

Hence, on 31^{st} night the show will cease to be profitable.

S5. Let a be the value of the certificates purchased by him in the 1^{st} year.

Common difference is Rs. 100. Then,

$$S_5 = 5250$$

We know that, $$S_5 = \frac{5}{2}[2a + (5-1)d]$$

$\Rightarrow$ $$5250 = \frac{5}{2}[2a + 4 \times 100]$$

$\Rightarrow$ $$\frac{5250 \times 2}{5} = 2a + 400$$

$\Rightarrow$ $$2100 = 2a + 400$$

$\Rightarrow$ $$2a = 2100 - 400$$

$\Rightarrow$ $$a = \frac{1700}{2} = 850$$

Hence, the value of certificate in the 1^{st} year = Rs. 850

$\therefore$ $$T_9 = a + (9-1)d$$

$\Rightarrow$ $$T_9 = 580 + 8 \times 100 = 850 = 1,650$$

which is the value of certificates purchased in the 9^{th} year.

S6. The age of the youngest student (a) = 8 years.

Common difference of their ages (d) = 4 months = $\frac{1}{3}$ year.

Sum of ages of all students (S_n) = 168 years.

Let n be the number of students in the class.

We know that, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 168 = \frac{n}{2}\left[2 \times 8 + (n-1)\cdot\frac{1}{3}\right]$$

$$\Rightarrow \quad 2 \times 168 = n\left[16 + \frac{(n-1)}{3}\right]$$

$$\Rightarrow \quad 336 = 16n + \frac{n(n-1)}{3}$$

$$\Rightarrow \quad 1{,}008 = 48n + n^2 - n$$

$$\Rightarrow \quad n^2 + 47n - 1{,}008 = 0$$

$$\Rightarrow \quad n^2 + 63n - 16n - 1{,}008 = 0$$

$$\Rightarrow \quad n(n + 63) - 16(n + 63) = 0$$

$$\Rightarrow \quad (n + 63)\,(n - 16) = 0$$

Either $\quad n + 63 = 0 \quad$ or $\quad n - 16 = 0$

$\Rightarrow \quad n = -63$ or $n = 16 \qquad$ [$n = -63$ (not possible)]

Hence, the number of students are 16.

S7. Initial deposit = Rs 10,000

Rate of interest = 5% p.a. S.I.

$$\text{Interest for one year} = \text{Rs.}\left(\frac{10{,}000 \times 5 \times 1}{100}\right) = \text{Rs. } 500$$

The amount in the account of the man in first, second, third, ..., years are Rs. 10,000, Rs. 10,500, Rs. 11,000, ...

Amount in 15^{th} years since deposit = Rs. $(10{,}000 + 14 \times 500)$

= Rs. 17,000

Amount after 20 years = Rs. $(10{,}000 + 20 \times 500)$

= Rs. 20,000.

S8. Gate receipt for the show on the first night = Rs. 6,500

Drop every succeeding night = Rs. –110

The show will cease profit on the night when receipt just = Rs. 1000

$$a = 6500,\ d = -110,\ T_n = 1000$$

$$T_n = a + (n-1)d$$

$$1000 = 6500 + (n-1)(-110)$$

$$110\,n = 6500 - 1000 + 110 = 5610$$

$$n = \frac{5610}{110} = 51$$

On 51^{st} night the show will cease to be profitable. After 51 nights there will be no show.

S9. Amount paid cash = Rs. 1,50,000

Remaining amount to be paid = Rs.(300000 – 150,000) = Rs. 1,50,000

$$\text{I installment} = \text{Rs. } 10{,}000 + (1{,}50{,}000)\frac{10}{100} = \text{Rs. } 25{,}000$$

$$\text{II installment} = \text{Rs. } 10{,}000 + (1{,}40{,}000)\frac{10}{100} = \text{Rs. } 24{,}000$$

$$\text{III installment} = \text{Rs. } 10{,}000 + (1{,}30{,}000)\frac{10}{100} = \text{Rs. } 23{,}000$$

$$\text{IV installment} = \text{Rs. } 10{,}000 + (1{,}40{,}000)\frac{10}{100} = \text{Rs. } 24{,}000$$

$$\text{Number of installments} = \frac{1{,}50{,}000}{10{,}000} = 15$$

$$a = 25000, 24000, 23000.........$$

$$d = -1000$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{15}{2}[2 \times 25{,}000 + (15-1)(-1000)]$$

$$= \frac{15}{2}[50{,}000 - 14000] = \frac{15}{2}(36000) = 15 \times 18000 = 2{,}70{,}000$$

Total amount paid for car = (Rs. 1,50,000 in cash + Rs. 2,70,000) = Rs. 4,20,000.

S10. Let the production in n^{th} year be a_n

a = production in 1st year, d = common difference

$\therefore$ $T_3 = 600$, $a + 2d = 600$ ... (i)

$a_7 = 700$, $a + 6d = 700$ ... (ii)

Solving (i) and (ii), we get

$d = 25$, $a = 550$ units

(a) 1st year production = a = 550 units

(b) the total production in 7 years.

$$S_n = \frac{n}{2}[2a + (n-1)d], \quad n = 7,$$

$$S_7 = \frac{7}{2}[2 \times 550 + (7-1)25] = \frac{7}{2}(1100 + 6 + 25)$$

$$= \frac{1250 \times 7}{2} = 625 \times 7 = 4375.$$

S11. Let the sides be $a - d$, a, $a + d$. are in AP

$a - d, a, a + d$ (given)

By pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(a + d)^2 = a^2 + (a - d)^2$$

$$a^2 + 2ad + d^2 = a^2 + a^2 - 2ad + d^2$$

$$a^2 = 4ad$$

$$a = 4d$$

Sides are $= a - d = 4d - d = 3d$, $a = 4d$, $a + d = 4d + d = 5d$

The sides are in the ratio 3 : 4 : 5.

S12. First term $= a =$ attendance of patients on first day

$\therefore$ $a = 750$

Common difference $d = -50$ (decline mean negative)

let on the n^{th} day, no. of patients = 0

$\therefore$ $a_n = 0, \quad a_n = a + (n - 1)d, \quad 0 = 750 + (n - 1)(-50)$

$$50(n - 1) = 750$$

$$n - 1 = \frac{750}{50} = 15,$$

$$n = 15 + 1 = 16$$

$\therefore$ on 16^{th} day, there will be no patient.

S13. Suppose the work was completed in n days when the workers started dropping. 4 workers dropped out every day except the first day. Therefore, the total number of workers who worked all the n days, is the sum of n terms of an A.P. with first term 150 and common difference -4, *i.e.*

$$\frac{n}{2}[2 \times 150 + (n - 1) \times (-4)] = n(152 - 2n)$$

Had the workers not dropped then the work should have finished in $(n - 8)$ days with 150 workers on each day. Therefore, the total number of workers who would have worked all the n day is $150(n - 8)$.

Therefore, $n(152 - 2n) = 150(n - 8)$

or $n^2 - n - 6000 = 0$

or $(n - 25)(n + 24) = 0$

or $n = 25$

Thus, the work was completed in 25 days.

S14.

$$a = \text{Rs. } 15625$$

$$\text{Depreciation} = 20\% \text{ of } 15625$$

$$= \frac{20}{100} \times 15625 = \text{Rs. } 3125$$

$$T_2 = 15625 - 3125 = \text{Rs. } 12500$$

$$\text{Common difference} = 12500 - 15625 = \text{Rs. } -3125$$

$$n = 5$$

$$T_n = a + (n - 1)\,d$$

$$T_5 = 15625 + (5 - 1)(-3125)$$

$$= 15625 - 12500 = \text{Rs. } 3125$$

Hence, estimated value at the end of 5 years = Rs. 3125.

S15. Given, Total cost of a scooter = \$ 12,000

Paid cash = \$ 6,000

Balance = \$ 6,000

Number of installments @ \$ 500 each = 12

$$\text{Interest on first installment} = \$\left(\frac{6000 \times 12 \times 1}{100}\right) = \$\,720$$

$$\text{First installment} = \$\,(500 + 720) = \$\,1220$$

$$\text{Interest on second installment} = \$\left(\frac{5500 \times 12 \times 1}{100}\right) = \$\,660$$

$$\text{Second installment} = \$\,(500 + 660) = \$\,1160$$

$$\text{Interest on third installment} = \$\left(\frac{5000 \times 12 \times 1}{100}\right) = \$\,600$$

$$\text{Third installment} = \$\,(500 + 600)$$

$$= \$\,1100 \text{ and so on}$$

Total amount paid in installments = \$ (1220 + 1160 + 1100 + ... to 12 terms)

Here, $a = 1220, \quad d = -60, \quad n = 12$

$$S = \frac{12}{2}\,[2 \times 1220 + (12 - 1)(-60)]$$

$$= 6\,(2440 - 11 \times 60)$$

$$= 6\,(2440 - 660)$$

$$= 6 \times 1780 = 10680$$

$$= \$\,10680$$

$\therefore$ Amount paid by farmer = \$ (6000 + 10680)

= \$ 16680.

S16. Given, Total cost of a scooter = \$ 22000

Paid cash = \$ 4000

Balance = \$ 18000

Number of installments @ \$ 1000 each = 18

$$\text{Interest on first installment} = \$\left(\frac{18000 \times 10 \times 1}{100}\right) = \$\,1800$$

First installment = \$ (1000 + 1800)

= \$ 2800

$$\text{Interest on second installment} = \$\left(\frac{17000 \times 10 \times 1}{100}\right) = \$\,1700$$

Second installment = \$ (1000 + 1700)

= \$ 2700

$$\text{Interest on third installment} = \$\left(\frac{16000 \times 10 \times 1}{100}\right) = \$\,1600$$

Third installment = \$ (1000 + 1600)

= \$ 2600 and so on

Total amount paid in installments = \$ (2800 + 2700 + 2600 + ... to 18 terms)

Here, $a = 2800, \quad d = -100, \quad n = 18$

$$S = \frac{18}{2}\,[2 \times 2800 + (18 - 1)(-100)]$$

= 9 (5600 – 1700)

$= 9 \times 3900 = 35100$

= \$ 35100

Amount paid by Ram = \$ (4000 + 35100)

= \$ 39100.

Date: 12/10/2021

Q1. Find the n^{th} term of the series: $1 + 2^2x + 3^2x^2 + 4^2x^3 + ...$

Q2. Find the sum to n terms of the series 1.3.6 + 2.5.9 + 3.7.12 + 4.9.15 + ...

Q3. Find the sum

(a) $5^2 + 6^2 + 7^2 + ... + 20^2$ (b) $2^3 + 4^3 + 6^3 + 8^3 + ... + 18^3$

Q4. Find the n^{th} term and the sum to n terms of the series: $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + ...$

Q5. If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, and their cubes respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.

Q6. Find the sum of first n terms of the series: 9 + 18 + 31 + 48 + ...

Q7. If f is a function satisfying $f(x + y) = f(x)f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{1}^{n} f(x) = 120$, find the value of n.

Q8. Find the n^{th} term of the series: $\frac{1}{3}+\frac{3}{9}+\frac{5}{27}+\frac{7}{81}+.......$

Q9. Show that:

$$\frac{1\cdot 2^2 + 2\cdot 3^2 + \cdots\cdots + n(n+1)^2}{1^2\cdot 2 + 2^2\cdot 3 + \cdots\cdots + n^2(n+1)} = \frac{3n+5}{3n+1}.$$

Q10. Find the sum of first n terms of the series: 10 + 17 + 26 + 37 + ...

Q11. Find the n^{th} and the sum to n terms of the series: 1 + 5 + 13 + 29 + ...

Q12. Find the sum to n terms of the series $1 + 5x + 9x^2 + 13x^3 + ...$ Hence, find the sum to infinity $|x| < 1$.

Q13. If the sum to infinity of the series $3 + 5r + 7r^2 + ...$ is $\frac{44}{9}$, find r.

Q14. If the sum of infinity of the series

$$1+(1+d)\left(-\frac{1}{3}\right)+(1+2d)\left(-\frac{1}{3}\right)^2+(1+3d)\left(-\frac{1}{3}\right)^3+...\text{ is }\frac{9}{16}\text{, find } d.$$

Q15. Find the sum of the infinite series: $\frac{1^2}{2}+\frac{3^2}{2^2}+\frac{5^2}{2^3}+\frac{7^2}{2^4}+......$

Q16. Find the sum of the series $1^2 + 5^2x + 9^2x^2 + 13^2x^3 + ... \infty$, where $|x| < 1$.

Q17. If the sum to infinity of the series: $3+(3+d)\frac{1}{4}+(3+2d)\frac{1}{4^2}+.........$ is $\frac{44}{9}$, find d.

Q18. If the sum to infinity of the series: $2 + 5x + 8x^2 + 11x^3 + ...$ is $\frac{35}{12}$, find x.

Q19. The sequence *N* of natural numbers is divided into classes as follows:

		1	2		
	3	4	5	6	
7	8	9	10	11	12

....

....

Show that the sum of the numbers in the n^{th} row is $n(2n^2 + 1)$.

Q20. Find the sum to *n* terms and infinite terms of the series: $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ...$

Date: 12/10/2021

S1. We have, $1 + 2^2x + 3^2x^2 + 4^2x^3 + ...$

This is not arithmetico-geometric series

First series: $1, 2^2, 3^2, 4^2 ...$ and G.P.: $1, x, x^2, x^3, ...$; Here, $a = 1$, $r = x$

n^{th} term of this series, $T_n = n^2$ ⇒ $T_n = a\,(r)^{n-1} = x^{n-1}$

∴ T_n of the given series = (T_n of first series) × (T_n of G.P.)

$\Rightarrow \quad T_n = (n^2)(x^{n-1}) = n^2x^{n-1}$

S2. 3^{rd} series: $6, 9, 12, ..., 3(n+1)$ $[T_n = 6 + (n-1)3]$

n^{th} term of the given series = (n^{th} term of the 1^{st} series) × (n^{th} term of the 2^{nd} series) × (n^{th} term of the 3^{rd} series)

$$\Rightarrow \quad T_n = n(2n+1).\,3(n+1) = 6n^3 + 9n^2 + 3n$$

Now,
$$S_n = \sum T_n = \sum(6n^3 + 9n^2 + 3n) = 6\sum n^3 + 9\sum n^2 + 3\sum n$$

$$= 6\left[\frac{n(n+1)}{2}\right]^2 + 9\left\{\frac{n(n+1)(2n+1)}{6}\right\} + 3\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{3}{2}\left[n^4 + 2n^3 + n^2\right] + \frac{3}{2}[2n^3 + 3n^2 + n] + \frac{3}{2}\left[n^2 + n\right]$$

$$= \frac{3}{2}\left[n^4 + 2n^3 + n^2 + 2n^3 + 3n^2 + n + n^2 + n\right]$$

$$= \frac{3}{2}\left[n^4 + 4n^3 + 5n^2 + 2n\right] = \frac{3}{2}n(n^3 + 4n^2 + 5n + 2).$$

S3. (a) Let $S = 5^2 + 6^2 + 7^2 + ... + 20^2$

Adding and subtracting $1^2 + 2^2 + 3^2 + 4^2$ in given series, we get

$$S = (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + ... + 20^2) - (1^2 + 2^2 + 3^2 + 4^2)$$

$$\Rightarrow \quad S = \left(\sum n^2\right)_{n=20} - \left(\sum n^2\right)_{n=4}$$

$$= \frac{20(20+1)(2.20+1)}{6} - \frac{4(4+1)(2.4+1)}{6} \quad \left[\because \sum n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{20 \times 21 \times 41}{6} - \frac{4 \times 5 \times 9}{6}$$

$$\Rightarrow \quad S = 2870 - 30 = 2840$$

(b) Let $S = 2^3 + 4^3 + 6^3 + ... + 18^3 = (2 \times 1)^3 + (2 \times 2)^3 + (2 \times 3)^3 + ... + (2 \times 9)^3$

$$= 2^3(1^3 + 2^3 + 3^3 + ... + 9^3) = 2^3 \left(\sum n^3\right)_{n=9} = 2^3 \left\{\frac{n(n+1)}{2}\right\}^2_{n=9}$$

$$= 8 \times \left(\frac{9(9+1)}{2}\right)^2 = 8 \times (45)^2 = 16200.$$

S4. The given series is $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + ...$ ... (i)

Let T_n be the n^{th} term of (i). Then,

$$T_n = 1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\text{Sum to } n \text{ terms} = S_n = \sum T_n = \frac{1}{3}\sum n^3 + \frac{1}{2}\sum n^2 + \frac{1}{6}\sum n$$

$$= \frac{1}{3}\left(\frac{n(n+1)}{2}\right)^2 + \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1}{6}\frac{n(n+1)}{2}$$

$$\Rightarrow \quad S_n = \frac{n(n+1)}{12}\left[n(n+1) + (2n+1)\right] = \frac{n(n+1)(n^2+3n+2)}{12}$$

$$= \frac{n(n+1)(n+1)(n+2)}{12} \quad \Rightarrow \quad S_n = \frac{n(n+1)^2(n+2)}{12}.$$

S5. We have, S_1 = Sum of first n natural $= \sum n = \frac{1}{2}n(n+1)$

S_2 = Sum of square of first n natural numbers $= \sum n^2 = \frac{n(n+1)(2n+1)}{6}$

And S_3 = Sum of cubes of first n natural numbers $= \sum n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Now, $$9S_2^2 = 9\left[\frac{n(n+1)(2n+1)}{6}\right]^2 = \frac{1}{4}n^2(n+1)^2(2n+1)^2 \quad ... \text{(i)}$$

And $$S_3(1 + 8S_1) = \frac{1}{4}n^2(n+1)^2\left[1 + 8 \cdot \frac{1}{2}n(n+1)\right] = \frac{1}{4}n^2(n+1)^2(2n+1)^2 \quad ... \text{(ii)}$$

From (i) and (ii), we get

$$9S_2^2 = S_3(1 + 8S_1).$$

S6. The given series is

$$9 + 18 + 31 + 48 + ...$$

$$(-) \quad 9 + 18 + 31 + 48 + ...$$

$$9 + (9 + 13 + 17 + ...)$$

The sequence of the difference between the two successive terms of this series is

$$(9, 13, 17, ...)$$

Clearly it is an AP with common difference 4. So let the n^{th} term of the given series is

$$T_n = an^2 + bn + c$$

Putting $n = 1, 2, 3$, we get

$$T_1 = a + b + c = 9$$
$$T_2 = 4a + 2b + c = 18$$
$$T_3 = 9a + 3b + c = 31$$

$$\begin{bmatrix} T_1 = 9 \\ T_2 = 18 \\ T_3 = 31 \end{bmatrix}$$

Solving these equations, we get $a = 2$, $b = 3$, $c = 4$

$$T_n = 2n^2 + 3n + 4$$

$$\text{Required sum} = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n}(2r^2 + 3r + 4) = 2\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r + 4\sum_{r=1}^{n} 1$$

$$= 2\left(\frac{n(n+1)(2n+1)}{6}\right) + 3\left(\frac{n(n+1)}{2}\right) + 4n = \frac{n}{6}[4n^2 + 15n + 35]$$

S7.

$$f(1) = 3, \ f(x + y) = f(x)\, f(y)$$
$$f(2) = f(1 + 1) = f(1) \,.\, f(1) = 3.3 = 9$$
$$f(3) = f(1 + 2) = f(1) \,.\, f(2) = 3.9 = 27$$
$$f(4) = f(1 + 3) = f(1) \,.\, f(3) = 3.27 = 81$$
$$S = \sum_{1}^{n} f(x) = f(1) + f(2) + f(3) + f(n) = 120$$

$\Rightarrow$ $3 + 9 + 27 + ...$ to n terms $= 120$

$\Rightarrow$ $$\frac{3(3^n - 1)}{3 - 1} = 120 \qquad \left[\because \ a = 3, r = 3; \ \ S = \frac{3(3^n - 1)}{3 - 1}\right]$$

$\Rightarrow$ $$\frac{3(3^n - 1)}{3 - 1} = 120$$

$\Rightarrow$ $$3^n - 1 = 120 \times \frac{2}{3} = 80$$

$$3^n = 80 + 1 = 81 = 3^4$$

$\Rightarrow$ $$n = 4$$

S8. We have $\frac{1}{3}+\frac{3}{9}+\frac{5}{27}+\frac{7}{81}+..........$

This is an arithmetico-geometric series with corresponding :

A.P. : 1, 3, 5, 7 ....and G.P. : $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, ...$

Here (for A.P.) $a = 1$, $d = (5 – 3) = 2$ | and for (G.P.) $a = \frac{1}{3}$, $r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$

$\Rightarrow$ T_n of (A.P.) $= a + (n – 1)d$ | And T_n (G.P.) $= ar^{n-1} = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{3^n}$

$\therefore$ T_n of A.G. series = (T_n of A.P.) $\times$ (T_n of G.P.)

$$= (2n-1)\left(\frac{1}{3^n}\right) = \frac{2n-1}{3^n}.$$

S9. n^{th} term of numerator is $n(n + 1)^2 = n^3 + 2n^2 + n$

$$\Rightarrow \quad T_n = n^3 + 2n^2 + n \quad \text{... (i)}$$

And n^{th} term of denominator is $n^2(n + 1) = n^3 + n^2$

$$\Rightarrow \quad T_n' = n^3 + n^2 \quad \text{... (ii)}$$

Now,

$$\text{L.H.S.} = \frac{1\cdot 2^2 + 2\cdot 3^2 + \cdots\cdots + n(n+1)^2}{1^2\cdot 2 + 2^2\cdot 3 + \cdots\cdots + n^2(n+1)}$$

$$= \frac{S}{S'} = \frac{\sum T_n}{\sum T_n'} = \frac{\sum (n^3 + 2n^2 + n)}{\sum (n^3 + n^2)} \quad \text{[Using (i) and (ii)]}$$

$$= \frac{\sum n^3 + 2\sum n^2 + \sum n}{\sum n^3 + \sum n^2}$$

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2 + 2\left(\frac{n(n+1)(2n+1)}{6}\right) + \left(\frac{n(n+1)}{2}\right)}{\left(\frac{n(n+1)}{2}\right)^2 + \left(\frac{n(n+1)(2n+1)}{6}\right)}$$

$$= \frac{\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}}{\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{\frac{n(n+1)}{12}\left\{3(n^2+n)+4(2n+1)+6\right\}}{\frac{n^2(n+1)^2}{4}\left\{3(n^2+n)+4(2n+1)\right\}}$$

$$= \frac{3n^2+3n+8n+4+6}{3n^2+3n+4n+2} = \frac{3n^2+11n+10}{3n^2+7n+2}$$

$$= \frac{(3n+5)(n+2)}{(3n+1)(n+2)} = \frac{3n+5}{3n+1} = \text{R.H.S.}$$

S10. The given series is 10 + 17 + 26 + 37 + ...

10 + 17 + 26 + 37 + ...

– 10 − 17 − 26 − 37 − ...

10 + (7 + 9 + 11 + ...)

The sequence of the difference between the two successive terms of this series are

(7, 9, 11, ...)

Clearly it is an AP with common difference 2. So, let the nth term of the give series is

$$T_n = an^2 + bn + c$$

Putting n = 1, 2, 3, we get

$T_1 = a + b + c = 10$	$T_1 = 10$
$T_2 = 4a + 2b + c = 17$	$T_2 = 17$
$T_3 = 9a + 3b + c = 26$	$T_3 = 26$

Solving these equations, we get $a = 1$, $b = 4$, $c = 5$

$$T_n = n^2 + 4n + 5$$

$$\text{Required sum} = \sum_{r=1}^{r=n} T_r = \sum_{r=1}^{n}(r^2+4r+5) = \sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r + 5\sum_{r=1}^{n} 1$$

$$= \frac{n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2} + 5n = \frac{n(n+1)(2n+1)}{6} + 2n(n+1) + 5n$$

$$= \frac{n}{6}[(n+1)(2n+1) + 12(n+1) + 30]$$

$$= \frac{n}{6}[2n^2 + 3n + 1 + 12n + 12 + 30] = \frac{n}{6}[2n^2 + 15n + 43].$$

S11. Let $S = 1 + 5 + 13 + 29 + ... + T_{n-1} + T_n$

And $S = (-)\,1 + 5 + 13 + ... + T_{n-2} + T_{n-1} + T_n$

$0 = 1 + 4 + 8 + 16 + ... + (T_{n-1}) - T_n$

$\Rightarrow \quad T_n = 1 + \{4 + 8 + 16 + ... + (n - 1) \text{ terms}\}$

$$= 1 + \frac{4(2^{n-1} - 1)}{2 - 1} \qquad \left[\because \text{ In G.P., } S_n = \frac{a(r^n - 1)}{r - 1}, r > 1\right]$$

$$= 1 + 4.2^{n-1} - 4 = 2^2 . 2^{n-1} - 3$$

$$T_1 = 2^{1+1} - 3 = 2^2 - 3$$

$$T_2 = 2^{2+1} - 3 = 2^3 - 3$$

$$T_3 = 2^{3+1} - 1 = 2^4 - 3$$

....

....

$$T_n = 2^{n+1} - 3$$

$S = T_1 + T_2 + T_3 + ... + T_n$ [On Adding]

$$= (2^2 - 3) + (2^3 - 3) + (2^4 - 3) + ... + (2^{n+1} - 3)$$

$$= 2^2(1 + 2 + 2^2 + ... + 2^{n-1}) - (3 + 3 + ... + n \text{ terms})$$

$$= 2^2 \cdot \frac{2^n - 1}{2 - 1} - 3n = 2^2.2^n - 4 - 3n$$

$$= 2^{n+2} - 3n - 4.$$

S12. We have $1 + 5x + 9x^2 + 13x^3 + ...$

This is an A.G. series with corresponding

A.P.: 1, 5, 9, 13...

and G.P.: $1, x, x^2, x^3, ...$

Here, $a = 1, \quad d = 4$ For corresponding A.P.

Here, $a = 1, r = x$ For corresponding G.P.

$\Rightarrow \quad T_n = 1 + (n - 1)4 = 4n - 3$

$\Rightarrow \quad T_n = a(r)^{n-1} = 1(x)^{n-1} = x^{n-1}$

$\therefore \quad T_n$ of A.G. series $= (4n - 3)(x^{n-1})$

$$S_n = 1 + 5x + 9x^2 + 13x^3 + ... + (4n - 7)x^{n-2} + (4n - 3)x^{n-1} \qquad ... (i)$$

Multiplying both sides of Eq. (i) by x, we get

$$xS_n = x + 5x^2 + 9x^3 + 13x^4 + ...+ (4n - 7)x^{n-1} + (4n - 3)x^{n-1} \qquad ... (ii)$$

Without touching 1st term of (i), we subtract (ii) from (i) as follows:

1st term of (ii) from 2nd term of (i) *i.e.*, $5x - x = 4x$

2nd term of (ii) from 3rd term of (i) *i.e.*, $9x^2 - 5x^2 = 4x^2$ and so on.

In this way, we get

$$(1-x)S_n = 1 + (5-1)x + (9-5)x^2 + (13-9)x^3$$
$$+ \ldots\ldots (4n-3-4+7)\, x^{n-1} - (4n-3)x^n$$
$$= 1 + 4x + 4x^2 + 4x^3 + \ldots\ldots + 4x^{n-1} - (4n-3)x^n$$
$$= 1 + 4\,(x + x^2 + x^3 + \ldots.. \text{ to } (n-1) \text{ terms}) - (4n-3)x^n$$

$$(1+x)S_n = 1 + 4\left\{\frac{x(1-x^{n-1})}{1-x}\right\} - (4n-3)x^n$$

$$\Rightarrow \qquad S_n = \frac{1}{1-x} + \frac{4x(1-x^{n-1})}{(1-x)^2} - \frac{4n-3}{1-3}x^n$$

Now, when $|x| < 1$ and $n \to \infty$, $x^n \to 0$

$$\therefore \qquad S = \frac{1}{1-x} + \frac{4x}{(1-x)^2} = \frac{1-x+4x}{(1-x)^2} = \frac{1+3x}{(1-x)^2}$$

S13. Let, $S = 3 + 5r + 7r^2 + 9r^3 + \ldots\ldots.. \text{ to } \infty$ ... (i)

Multiplying by r in equation 1[st]

$rS = 3r + 5r^2 + 7r^3 + \ldots\ldots..$ ... (ii)

Without touching 1[st] term of (i), we subtract (ii) from (i) as follows:

1[st] term of (ii) from 2[nd] term of (i) *i.e.*, $5x - x = 4x$

2[nd] term of (ii) from 3[rd] term of (i) *i.e.*, $9x^2 - 5x^2 = 4x^2$ and so on.

In this way, we get

$$(1-r)S = 3 + (5-3)r + (7-5)r^2 + (9-7)r^3 + \ldots$$

$$\Rightarrow \qquad (1-r)S = 3 + 2\,[r + r^2 + r^3 + \ldots] + \ldots$$

$$\Rightarrow \qquad (1+r)S = 3 + 2\,.\,3 + 2\cdot\frac{r}{1-r} = \frac{3-r}{1-r} \qquad \left[\because S_\infty = \frac{1}{1-r}\right]$$

$$\Rightarrow \qquad S = \frac{3-r}{(1-r)^2} \quad \Rightarrow \quad \frac{44}{9} = \frac{3-r}{(1-r)^2} \qquad \left[\because\ S = \frac{44}{9} \text{ given}\right]$$

$$\Rightarrow \qquad 9(3-r) = 44(1-2r+r^2) \Rightarrow \quad 44r^2 - 79r + 17 = 0$$

$$\Rightarrow \qquad r = \frac{79 \pm \sqrt{(-79)^2 - 4\times 44\times 17}}{2\times 44}$$

$$= \frac{79 \pm \sqrt{3249}}{88} = \frac{79 \pm 57}{88} = \frac{17}{11} \text{ or } \frac{1}{4}$$

Rejecting $r = \dfrac{17}{11}$, $[\because\ |r| < 1]$

Since the given series is convergent only when $|r| < 1$, we have $r = \dfrac{1}{4}$.

S14. We have,

$$S = 1 + (1 + d)\left(-\frac{1}{3}\right) + (1+2d)\left(-\frac{1}{3}\right)^2 + (1+3d)\left(-\frac{1}{3}\right)^3 + \ldots\ldots \quad \ldots \text{(i)}$$

Multiplying both sides of (i) by $\left(-\frac{1}{3}\right)$, we get

$$-\frac{1}{3}S = \left(-\frac{1}{3}\right) + (1+d)\left(-\frac{1}{3}\right)^2 + (1-2d)\left(-\frac{1}{3}\right)^3 + (1+3d)\left(-\frac{1}{3}\right)^4 + \ldots \quad \text{(ii)}$$

Without touching 1st term of (i) we subtract (ii) from (i) as follows:

1st term of (ii) from 2nd term of (i) *i.e.*,

$$(1+d)\left(-\frac{1}{3}\right) - \left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)d$$

2nd term of (ii) from 3rd term of (i) *i.e.*,

$$(1+2d)\left(-\frac{1}{3}\right)^2 - (1+d)\left(-\frac{1}{3}\right)^2 = \left(-\frac{1}{3}\right)^2 d \text{ and so on}$$

In this way subtracting (ii) from (i), we get

$$\left(1+\frac{1}{3}\right)S = 1 + d\left[\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 + \ldots\infty\right]$$

$$= 1 + d\left(\frac{-\frac{1}{3}}{1+\frac{1}{3}}\right) = 1 + d\left(-\frac{1}{4}\right)$$

$$\Rightarrow \quad S = \left[1 + d\left(-\frac{1}{4}\right)\right]\left(\frac{3}{4}\right) = \frac{3}{4} - \frac{3}{16}d$$

$$\Rightarrow \quad \frac{9}{16} = \frac{3}{4} - \frac{3}{16}d \Rightarrow \frac{3}{16}d = \frac{3}{4} - \frac{9}{16} = \frac{3}{16} \Rightarrow d = 1.$$

S15. Let

$$S = \frac{1^2}{2} + \frac{3^2}{2^2} + \frac{5^2}{2^3} + \frac{7^2}{2^4} + \ldots\ldots \quad \ldots \text{(i)}$$

$$\frac{1}{2}S = \frac{1^2}{2^2} + \frac{3^2}{2^3} + \frac{5^2}{2^4} + \ldots \quad \left[\text{Multiplying by common ratio, } \frac{1}{2}\right] \quad \text{.. (ii)}$$

$$\Rightarrow \quad S - \frac{1}{2}S = \frac{1^2}{2} + \frac{3^2 - 1^2}{2^2} + \frac{5^2 - 3^2}{2^3} + \frac{7^2 - 5^2}{2^4} + \ldots\ldots. \quad \ldots \text{(iii)}$$

$$\Rightarrow \quad \frac{1}{2}S = \frac{1^2}{2} + 8\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right)^3 + 24\left(\frac{1}{2}\right)^4 + \ldots\ldots$$

$$\Rightarrow \quad \frac{1}{2}S - \frac{1}{2} = 8\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right)^3 + 24\left(\frac{1}{2}\right)^4 + \ldots\ldots \quad \ldots \text{(iv)}$$

Here, 8, 16, 24, ... is an A.P. and $\left(\frac{1}{2}\right)^2$, $\left(\frac{1}{2}\right)^3$, $\left(\frac{1}{2}\right)^4$ ... is a G.P.

Multiplying (iv) by $\frac{1}{2}$, we get

$$\frac{1}{2}\times\frac{1}{2}S - \frac{1}{2}\times\frac{1}{2} = 8\left(\frac{1}{2}\right)^3 + 16\left(\frac{1}{2}\right)^3 + 24\left(\frac{1}{2}\right)^5 + \ldots\ldots$$

Subtracting (iv) from (iii), we get

$$\frac{1}{2}S - \frac{1}{2} - \frac{1}{4}S + \frac{1}{4} = 8\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right)^3 + 8\left(\frac{1}{2}\right)^4 + \ldots\ldots..$$

$$\Rightarrow \quad \frac{S}{4} - \frac{1}{4} = 8\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \ldots\ldots\right] = 8\left[\frac{\left(\frac{1}{2}\right)^2}{1-\frac{1}{2}}\right]$$

$$\Rightarrow \quad \frac{1}{4}S - \frac{1}{4} = 8\times\frac{1}{2} \quad \Rightarrow \quad \frac{1}{4}S = 4 + \frac{1}{4} = \frac{17}{4} \quad \Rightarrow \quad S = 17$$

S16. Let

$$S = 1^2 + 5^2x + 9^2x^2 + 13^2x^3 + \ldots. \infty \quad \ldots \text{(i)}$$

$$xS = 1^2x + 5^2x^2 + 9^2x^3 + \ldots\ldots \infty \quad \ldots \text{(ii)}$$

$$\Rightarrow \quad S - xS = 1^2 + (5^2 - 1^2)x + (9^2 - 5^2)x^2 + (13^2 - 9^2)x^3 + \ldots.. \infty$$

$$\Rightarrow \quad (1-x)S = 1 + 24x + 56x^2 + 88x^3 + \ldots\ldots \infty \quad \ldots \text{(iii)}$$

(iii) is an arithmetico-geometric series as 24, 56, 88... is an A.P. and x, x^2, x^3, ... is a G.P.

Multiplying (iii) by x, we get

$$x(1-x)S - x = 24x^2 + 56x^3 + 88x^4 + \ldots\ldots \infty \quad \ldots \text{(iv)}$$

Subtracting (iv) from (iii), we get

$$(1-x)S - 1 - x(1-x)S + x = 24x + 32x^3 + \ldots. \infty$$

$$\Rightarrow \quad (1-x)S(1-x) - 1 + x = 24x + 32[x^2 + x^4 + \ldots.. \infty]$$

$$\Rightarrow \quad (1-x)^2 S - (1-x) = 24x + \frac{32x^2}{1-x} \quad [\because \ |x| < 1]$$

$$\Rightarrow \quad (1-x)^2 S = 1 + 23x + \frac{32x^2}{1-x} = \frac{1+22x+9x^2}{(1-x)^3}$$

$$\Rightarrow \quad S = \frac{1+22x+9x^2}{(1-x)^3}.$$

S17. We have $S = 3 + (3 + d)\ \frac{1}{4}\ + (3 + 2d)\ \frac{1}{4^2} + ...$ ... (i)

Multiplying both sides of (i) by $\frac{1}{4}$, we get

$$\frac{1}{4}S = \frac{3}{4} + (3+d)\left(\frac{1}{4}\right)^2 + (3+2d)\left(\frac{1}{4}\right)^3 + ... \quad ...\text{(ii)}$$

Without touching 1st term of (i) we subtract (ii) from (i) as follows:

1st term of (ii) from 2nd term of (i) *i.e.*, $(3+d)\frac{1}{4} - \frac{3}{4} = \frac{1}{4}(3+d-3) = \frac{d}{4}$

2nd term of (ii) from 3rd term of (i) *i.e.*, $(3+2d)\left(\frac{1}{4}\right)^2 - (3+d)\left(\frac{1}{4}\right)^2$

$$= \left(\frac{1}{4}\right)^2 (3+2d-3-d) = d\left(\frac{1}{4}\right)^2 \text{ and so on.}$$

In this way, subtracting (ii) from (i), we get

$$\left(1-\frac{1}{4}\right)S = 3 + d\left\{\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ...\right\}$$

$$\Rightarrow \quad \frac{3}{4}S = 3 + d\left\{\frac{\frac{1}{4}}{1-\frac{1}{4}}\right\} = 3 + d\left(\frac{4}{3}\times\frac{1}{4}\right) = 3 + \frac{d}{3} \qquad \left[\because S_\infty = \frac{a}{1-r}\right]$$

$$\Rightarrow \quad S = 4 + \frac{4}{9}d$$

$$\Rightarrow \quad \frac{44}{9} = 4 + \frac{4}{9}d \qquad \left[\because S_\infty = \frac{a}{1-r} \text{ given}\right]$$

$$\Rightarrow \quad \frac{4}{9}d = \frac{44}{9} - 4 = \frac{44-36}{9} = \frac{8}{9}$$

$$\Rightarrow \quad \frac{44}{9} = 4 + \frac{4}{9}d$$

$$\Rightarrow \quad \frac{4}{9}d = \frac{44}{9} - 4 = \frac{44-36}{9} = \frac{8}{9}$$

$$\Rightarrow \quad d = \frac{8}{9}\times\frac{9}{4} = 2$$

Hence, $d = 2$.

S18. Let, $S = 2 + 5x + 8x^2 + 11x^3 + \ldots \infty$ ... (i)

Multiplying both sides by x, we get

$xS = 2x + 5x^2 + 8x^3 + 11x^4 + \ldots \infty$ ...(ii)

Without touching 1st term of (i), we subtract (ii) from (i) as follows:

1st term of (ii) from 2nd term of (i) *i.e.*, $5x - 2x = 3x$

2nd term of (ii) from 3rd term of (i) *i.e.*, $8x^2 - 5x^2 = 3x^2$ and so on

In this way we get

$$(1 - x)S = 2 + 3x + 3x^2 + 3x^3 + \ldots \infty$$

$$= 2 + 3(x + x^2 + x^3 + \ldots \infty) = 2 + \left(\frac{x}{1-x}\right) = \frac{2+x}{1-x}$$

$\Rightarrow$ $S = \dfrac{2+x}{(1-x)^2}$ $\Rightarrow$ $\dfrac{35}{12} = \dfrac{2+x}{(1-x)^2}$

$\Rightarrow$ $35(1 - x)^2 = 12(2 + x)$ $\Rightarrow$ $35 + 35x^2 + 70x - 24 - 12x = 0$

$\Rightarrow$ $35x^2 - 82x + 11 = 0$

$\Rightarrow$ $(7x + 1)(5x - 11) = 0$ $\Rightarrow$ $x = -\dfrac{1}{7}, \dfrac{11}{5}$ $\left[\text{Reject } \dfrac{11}{5} \text{ as } \dfrac{11}{5} > 1\right]$

hence $x = -\dfrac{1}{7}$

S19. Number of terms in the first row = 2

Number of terms in the second row = 4

Number of terms in the third row = 6

....

....

Number of terms in the n^{th} row = $2n$

Now, number of terms upto the end of the n^{th} row

$$= 2 + 4 + 6 + \ldots + 2n$$

$$= 2(1 + 2 + 3 + \ldots + n) = 2\left\{\sum n\right\} = 2\left(\frac{n(n+1)}{2}\right) = n^2 + n \quad \ldots(i)$$

Also, number of terms upto the end of the $(n-1)^{th}$ row $= (n-1)^2 + (n - 1)$ [Using (i)]

$\therefore$ Sum of terms in the n^{th} row = Sum of terms upto the end of the n^{th} row – Sum of terms upto the end of the $(n-1)^{th}$ row.

$$= \sum_{k=1}^{n} k - \sum_{k}^{n-1} k = \frac{(n^2+n)(n^2+n+1)}{2} - \frac{(n^2-n)(n^2-n+1)}{2}$$

$$= \frac{n}{2}\left[(n+1)(n^2+n+1)-(n-1)(n^2-n+1)\right]$$

$$= \frac{n}{2}\left[4n^2+2\right] = n(2n^2+1).$$

S20. We have, $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots$

Each term of the denominator of the given series is fomed by multiplication of the corresponding terms of the following two series :

1st series: $2, 5, 8, \ldots\ldots, (3n-1)$. $[T_n = 2+(n-1)3]$

2nd series: $5, 8, 11, \ldots\ldots, (3n+2)$. $[T_n = 5+(n-1)3]$

n^{th} term of the given series = $\frac{1}{(n^{th}\text{ term of }1^{st}\text{ series})\times(n^{th}\text{ term of }2^{nd}\text{ series})}$

$\Rightarrow$ $T_n = \frac{1}{(3n-1)(3n+2)} = \frac{1}{3}\left[\frac{1}{3n-1}-\frac{1}{3n+2}\right]$ [By partial fractions]

Putting $n = 1, 2, 3, \ldots, n$ successively, we get

$$T_1 = \frac{1}{3}\left[\frac{1}{2}-\frac{1}{5}\right]$$

$$T_2 = \frac{1}{3}\left[\frac{1}{2}-\frac{1}{8}\right]$$

$$T_3 = \frac{1}{3}\left[\frac{1}{8}-\frac{1}{11}\right]$$

....

....

$$T_n = \frac{1}{3}\left[\frac{1}{3n-1}-\frac{1}{3n+2}\right]$$

Adding vertically, we get

$$S_n = \frac{1}{3}\left(\frac{1}{2}-\frac{1}{3n+2}\right) = \frac{1}{6}-\frac{1}{3(3n+2)} = \frac{3n+2-2}{6(3n+2)} = \frac{n}{2(3n+2)}$$

$\Rightarrow$ $S_n = \frac{1}{2\left(3+\frac{2}{n}\right)}.$

Now, as $n\to\infty,\ \frac{2}{n}\to 0 \Rightarrow S_\infty = \frac{1}{2(3+0)} = \frac{1}{6}.$